INNOVATION CONFIGURATION

Empirically-Validated Mathematics Instruction for Grades K-5

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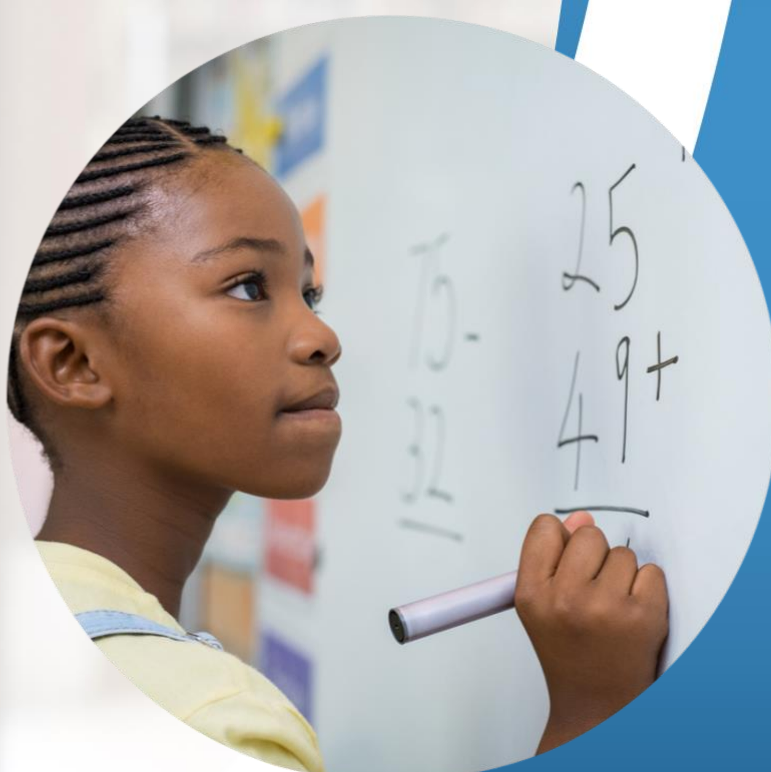

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Innovation Configuration for Empirically Validated Mathematics Instruction for Grades K-5

This paper presents an innovation configuration (IC) matrix to guide teacher preparation professionals in supporting pre-service and in-service educators in building scientifically aligned knowledge and the ability to use empirically validated instructional practices in mathematics for K-5 students at risk or identified with a disability. This matrix appears in the Appendix. An IC is a tool for identifying and describing the major components of a practice or innovation. With the implementation of any innovation comes a continuum of implementation configurations, ranging from non-use to the ideal. In this IC, a rating of 3 represents the strongest implementation of an individual component, but it should not be interpreted to mean that every course or preparation program must devote equal time or emphasis to every component. Program contexts, including course sequence, candidate background knowledge, field placement expectations, instructional time, and local priorities, may shape how the components are introduced, emphasized, and revisited. Thus, the IC is intended to guide coherent program design and self-assessment rather than function as a uniform checklist in which all components must receive identical instructional weight.

ICs are organized around two dimensions: essential components and degree of implementation (Hall & Hord, 1987; Roy & Hord, 2004). Essential components of the IC, along with descriptors and examples to guide application of the criteria to coursework, standards, and classroom practices, are listed in the rows of the far-left column of the matrix. Several levels of implementation are defined in the top row of the matrix. For example, the absence of a mention of the essential component is the lowest level of implementation and would receive a score of zero. Increasing implementation levels receive progressively higher scores.

ICs have been used in the development and implementation of educational innovations for at least 30 years (Hall & Hord, 1987; 2001; Hall et al., 1975; Roy & Hord, 2004). Experts at a national research center originally developed these tools, which are used for professional development (PD) within the Concerns-Based Adoption Model. The tools have also been used for program evaluation (Hall & Hord, 2001; Roy & Hord, 2004).

Use of this tool to evaluate course syllabi can help teacher preparation leaders ensure they emphasize empirically validated instructional practices in mathematics. The IC included in the Appendix of this paper is designed for teacher preparation programs, but it can be modified as an observation tool for professional development.

Importance of Mathematics

Mathematical proficiency serves as a gateway to academic and career opportunities, grounding students in problem-solving, logical reasoning, and quantitative literacy that extend across nearly every professional field (Maass et al., 2019; National Mathematics Advisory Panel, 2008). As students progress through school, mathematical competence continues to shape long-term outcomes, influencing college access and employment prospects (Lee, 2012), particularly as mathematics-intensive occupations expand at rates far exceeding overall job growth (National Mathematics Advisory Panel, 2008). However, national achievement data reveal persistent and worsening underperformance in mathematics. Data from the National Assessment of Educational Progress (NAEP), administered biennially since 1990, show that mathematics proficiency rates have remained stubbornly low for over two decades, with particularly pronounced declines following the COVID-19 pandemic (National Center for Education Statistics [NCES], 2026).

The 2024 NAEP results indicate that only 39% of fourth-grade students and 28% of eighth-grade students performed at or above the proficient level in mathematics, representing declines of two percentage points for fourth grade and six percentage points for eighth grade since 2019, the last NAEP administration prior to the COVID-19 pandemic and its disruption to schooling (NCES, 2026). Outcomes for students with disabilities were substantially lower. In 2024, only 15% of fourth-grade students with disabilities and 9% of eighth-grade students with disabilities achieved proficient performance, underscoring persistent achievement gaps across grade levels (NCES, 2026). The broader 2024 NAEP mathematics results further suggest that pandemic-related disruptions intensified existing patterns of academic decline. Twelfth-grade students declined by 3 points in mathematics since 2019, and the lowest-performing 12th-grade students, those scoring at the 10th percentile, showed declines that predated the pandemic but worsened between 2019 and 2024 (National Assessment Governing Board, 2025). These inequitable outcomes reflect many contributing variables, but the characteristics of mathematics instruction students receive in school remain among the most actionable levers available to educators and policymakers.

These sustained patterns of underperformance, exacerbated by pandemic-related disruptions, demonstrate that mathematics difficulties represent a persistent and widespread challenge requiring empirically validated instruction tailored to individual learning needs. Over decades of research, investigators have identified a set of empirically validated instructional practices that effectively support students with or at risk of mathematics difficulties when implemented with fidelity (Fuchs et al., 2021; Gersten et al., 2009a; Powell et al., 2025). Notably, the majority of this literature has focused on students with or at risk for mathematics difficulties, including students with learning disabilities in mathematics, students in other high-incidence disability categories who experience mathematics-related challenges, and students without identified disabilities who demonstrate persistent low performance in mathematics (Myers et al., 2021; Powell et al., 2019, 2021;Stevens et al., 2018). This evidence base informs the present IC and reflects the structure of the broader literature, where the distinction between formal disability identification and persistent low performance is often not sharply drawn. Across these groups, research indicates that many of the same empirically validated practices are effective (Jitendra et al., 2018; Myers et al., 2021). In instances where evidence indicates that a practice is also effective for students with low-incidence disabilities, this document notes that connection within the relevant section. This IC organizes these research-validated practices into a practical framework for K–5 mathematics instruction, providing educators with clear guidance on essential instructional components and benchmarks for high-quality implementation.

What Knowledge and Skills Do Teachers Need to Teach Mathematics?

Teachers' knowledge and expertise primarily determine the effectiveness of mathematics instruction in elementary schools. Research on teacher characteristics in the general student population has consistently shown that teacher qualifications are significantly associated with gains in students' mathematics achievement (Myers et al., 2022a; Wayne & Youngs, 2003). However, many teachers, including those with mathematics-related credentials, lack the specialized preparation needed to teach mathematics effectively to students with persistent mathematics difficulties. This gap has been documented in the special education literature for decades (Parmar & Cawley, 1997). Shulman (1986, 2000) identified seven categories of teacher knowledge, including content knowledge, general pedagogical knowledge, and pedagogical content knowledge, which integrates subject matter and instructional methods to make content comprehensible to learners. Snow et al. (2005) further conceptualized teacher knowledge as developing along a progressive differentiation model comprising five levels: declarative knowledge, situated procedural knowledge, stable procedural knowledge, expert adaptive knowledge, and reflective organized knowledge. Each level corresponds to a stage in a teacher's career, from preservice to master teacher. This IC focuses on the first two levels of this model, declarative knowledge and situated procedural knowledge, which form the foundational knowledge and skills that preservice preparation programs are responsible for developing in candidates preparing to teach mathematics to students with and at risk for mathematics difficulties.

Building on this taxonomy, Ball et al. (2008) developed a practice-based theory of mathematical knowledge for teaching, identifying empirically discernible subdomains within Shulman's broader categories. These subdomains include specialized content knowledge unique to teaching mathematics, as well as knowledge of content and students. These areas are not merely theoretical; they reflect the actual mathematical reasoning required when teachers interpret student errors, evaluate representations, and sequence tasks to build understanding. Myers et al. (2022a), using latent class analysis, found that adolescents at risk for mathematics difficulties who were assigned to novice and early-career, traditionally prepared mathematics majors achieved significantly lower mathematics learning gains than peers assigned to teachers with greater experience and stronger content and pedagogical knowledge. This evidence indicates that subject-matter preparation alone, without the depth of instructional experience developed over time, is insufficient for supporting students who struggle most. Collectively, these findings demonstrate that content preparation alone is inadequate; effective mathematics teaching, particularly for students with or at risk for mathematics difficulties, requires a specialized and multidimensional knowledge base developed through intentional preparation. Contributions from several converging scientific disciplines inform this knowledge base.

Cognitive psychology has clarified the mental processes underlying mathematical learning, including the role of working memory in manipulating numerical information, the conditions that lead to cognitive overload, and the distinction between procedural fluency and conceptual understanding. Both procedural fluency and conceptual knowledge must be addressed in instruction (Geary, 2004; Sweller, 1988). Neuroscience has identified the neural systems involved in numerical cognition and magnitude processing, provided biological explanations for persistent difficulties in mathematics, and demonstrated that targeted, evidence-based instruction can produce measurable changes in brain function and performance (Butterworth et al., 2011; Price & Ansari, 2013). The discipline of mathematics itself offers an essential content framework that specifies hierarchical relationships among concepts, the precision of mathematical language, and the sequence in which ideas should be developed to support subsequent learning (National Mathematics Advisory Panel, 2008; National Research Council, 2001). Education research, particularly in special education, translates these disciplinary findings into instructional designs that can be implemented reliably across diverse classroom contexts, evaluated through rigorous experimental and quasi-experimental designs, and replicated with students representing a wide range of learning profiles (Gersten et al., 2009a; Witzel et al., 2024). No single discipline provides a complete perspective; rather, their integration constitutes the science of mathematics instruction.

For teachers working with students who struggle with mathematics, particularly those serving students with disabilities, content knowledge and general pedagogical knowledge are necessary but not sufficient (Myers et al., 2022a). These teachers require an additional layer of technical instructional knowledge, including the ability to intensify instruction systematically, use student performance data to individualize instructional decisions, and implement evidence-based practices with fidelity across varying levels of student need (Fuchs et al., 2017; Powell et al., 2024; Sayeski et al., 2023; Witzel et al., 2026). Parmar and Cawley (1997) identified this imperative nearly three decades ago, arguing that teachers of students with learning disabilities in mathematics must draw on both the content standards advanced by the mathematics education community and the specialized instructional competencies delineated by the special education field. This dual demand has not diminished; rather, it has intensified as the research base has become more precise regarding the requirements for effective instruction for this population. Students with mathematics difficulties often have profiles that complicate instructional planning, such as co-occurring reading difficulties, working memory constraints, and inconsistent fact retrieval. Each of these factors requires teachers to make principled adjustments to pacing, representation, and practice design, rather than simply delivering grade-level content through standard pedagogical approaches (Geary, 2004). The knowledge required to make these adjustments is specialized, learnable, and teachable through well-designed preparation.

Despite the extensive body of accumulated knowledge, translating research into consistent classroom practice remains a persistent challenge in mathematics education. Although research has significant potential to improve outcomes for students with mathematics difficulties, this potential has not been fully realized in teacher preparation or in-service professional learning contexts. Parmar and Cawley (1997) documented the misalignment between the content of preparation programs and the dual demands of mathematics and special education standards nearly 30 years ago, and subsequent evidence indicates that this gap has persisted. Many teachers, particularly those preparing to serve students with disabilities, report entering classrooms without sufficient preparation in evidence-based instructional strategies for mathematics. The gap between research-supported practices and those routinely implemented remains consequential (Brownell et al., 2010; Leko et al., 2012). For students with disabilities and those experiencing mathematics difficulties, this gap is not solely an academic concern. Access to appropriately designed, evidence-based instruction is legally mandated under IDEA (2004) and is essential for closing persistent achievement disparities. Understanding the research base's recommendations and their rationale is, therefore, a prerequisite for teachers, teacher educators, and policymakers seeking to improve mathematics outcomes at scale.

Several professional groups and federal agencies have outlined what mathematics teachers need to know and be able to do. Their guidelines reflect research on effective teaching. The Council for Exceptional Children (CEC), the National Council of Teachers of Mathematics (NCTM), and the CEEDAR Center each provide frameworks for successful mathematics instruction. Teacher preparation programs use these to design and strengthen curricula. These frameworks all focus on proven, research-based teaching practices. The present IC fits within this field of standards and ongoing gaps between research and practice.

The following sections synthesize the research and policy context that informs evidence-based mathematics instruction for students with special education needs in Grades K–5. Section 1.0 examines eight dimensions of this context in detail, including the history of mathematics education policy and the math wars, the foundational science of empirically validated practices, landmark evidence syntheses, IES practice guide recommendations, federal legislative mandates, the Common Core State Standards, professional organization standards, and the influence of public media on perceptions of mathematics instruction. Across these dimensions, the central question remains: what does the accumulated evidence indicate about effective mathematics instruction, and what knowledge and skills teachers need to implement it for all students, including those with disabilities and those experiencing persistent mathematics difficulties? A persistent tension in this landscape is that elementary teacher preparation is often shaped by mathematics education traditions that do not always fully align with the empirical literature in special education, particularly regarding explicit, systematic instruction for students with disabilities and those experiencing persistent mathematics difficulties (Cohen et al., 2025). This creates a preparation gap that the present IC is designed to address. Addressing this question with clarity and purpose is essential for advancing equitable and effective mathematics education.

1.0. Influences on Mathematics Policy and Practice in the United States

Understanding the policy and research landscape that has shaped mathematics instruction in the United States is essential for teacher preparation programs seeking to ground their curricula in empirical evidence and professional consensus. This section traces eight intersecting dimensions of that landscape: the foundational science of empirically validated practices; the historical arc of mathematics education reform, including the math wars; landmark evidence syntheses; practice guide recommendations from the Institute of Education Sciences; federal legislative mandates; nationwide curricular initiatives; standards advanced by professional organizations; and the role of public media in shaping perceptions of mathematics instruction. Taken together, these dimensions define the context in which effective, specially designed mathematics instruction for students with disabilities must be situated and understood.

1.1. Empirically Validated Practices and Specially Designed Instruction

Empirically validated practices are instructional approaches, strategies, and interventions whose effectiveness has been established through methodologically rigorous research measuring student outcomes (Cook et al., 2015). In mathematics instruction and intervention, empirically validated practices represent the cumulative findings of several scientific disciplines, each contributing at complementary levels of analysis. Cognitive psychology offers foundational insights into how students acquire, store, and retrieve mathematical information, including research on working memory limitations, the distinction between procedural fluency, the capacity to execute procedures flexibly, accurately, efficiently, and appropriately (Burns et al., 2015), and conceptual understanding, comprehending mathematical concepts, operations, and relations (Byrnes & Wasik, 1991; Hiebert, 2013; National Research Council, 2001). Both constructs are bidirectionally related and must be addressed in instruction (Geary, 2004; Rittle-Johnson et al., 2015; Sweller, 1988).

Neuroscience extends this understanding by identifying neural systems involved in numerical cognition, magnitude processing, and arithmetic fact retrieval. This research provides a biological basis for persistent mathematics difficulties and demonstrates that targeted instruction can produce measurable improvements in both performance and brain function (Butterworth et al., 2011; Hughes et al., 2023; Price & Ansari, 2013). The discipline of mathematics offers a content-knowledge framework, including the hierarchical structure of concepts and procedures, the interrelationships among domains, and the precise language required to represent and communicate mathematical ideas. This framework informs both the content and sequence of instruction (National Mathematics Advisory Panel, 2008; National Research Council, 2001). Educational research, particularly in special education, translates these disciplinary findings into instructional designs that can be implemented reliably in school settings, evaluated through randomized controlled trials and single-case designs, and replicated across diverse student populations (Gersten et al., 2009a; Witzel et al., 2024).

Specially designed instruction (SDI) in mathematics applies this cross-disciplinary knowledge base to address the individualized needs of students with disabilities. According to IDEA (2004, 34 C.F.R. §300.39[b][3]), SDI involves adapting content, methodology, or instructional delivery to meet the unique needs of students with disabilities and to ensure their access to the general education curriculum. Operationalizing this mandate requires special educators to provide support across varying levels of intensity, from access supports and intensified instruction to intensive intervention, each aligned with a student's Individualized Education Program (IEP) goals and present levels of academic achievement and functional performance (PLAAFP: Sayeski et al., 2023). In practice, SDI in mathematics draws on psychological research to inform decisions about pacing, practice scheduling, and cognitive load; utilizes neuroscientific findings to understand the profiles of students with dyscalculia or co-occurring reading and mathematics disabilities; applies mathematical content knowledge to ensure instructional targets are accurate and appropriately sequenced; and leverages education research to select and implement specific empirically validated practices with fidelity (Witzel et al., 2026). The integration of these disciplines enables educators to move from a general commitment to supporting students with mathematics difficulties to making specific, empirically validated instructional decisions regarding content, pedagogy, and assessment of effectiveness. As Witzel et al. (2024) emphasized, the field of special education has consistently prioritized outcome data, and the identification of empirically validated practices has been shaped by rigorous research across these disciplines.

1.2. History of Mathematics Education Reform and the Math Wars

Mathematics instruction in the United States has undergone repeated and often contentious reform cycles over the past century, shaped by shifting political priorities, evolving learning theories, and an accumulating body of empirical research. Through the mid-20th century, mathematics education emphasized procedural fluency and rote memorization of facts and algorithms (Kilpatrick et al., 2001). Following the Soviet launch of Sputnik in 1957, the federal government directed substantial resources toward modernizing mathematics curricula, producing the "*New Math*" movement of the 1960s, which prioritized abstract mathematical structures and set theory. This approach proved difficult for teachers to implement and was broadly abandoned by the 1970s, giving way to a renewed focus on foundational skills. The 1983 federal report *A Nation at Risk* intensified this scrutiny, characterizing declining student achievement as a threat to national competitiveness and creating sustained political pressure for systemic reform in mathematics education (National Commission on Excellence in Education, 1983). The pendulum shifted again in 1989, when the National Council of Teachers of Mathematics (NCTM) released its Curriculum and Evaluation Standards for School Mathematics, which advanced a constructivist vision of mathematics learning centered on student inquiry, conceptual understanding, problem solving, and mathematical discourse (NCTM, 1989). These standards were influential but also immediately contested, setting the stage for what became known as the "*math wars*."

The term "*math wars*" refers to the ongoing and sometimes acrimonious debate between proponents of constructivist, inquiry-based approaches to mathematics instruction and advocates of systematic, explicit instruction grounded in empirical research. Constructivism originated as a theory of how individuals construct knowledge through experience (Piaget, 1952) and was later adopted as an instructional approach in which students discover mathematical relationships through exploration rather than direct teaching (Elliott et al., 2000). Supporters of constructivist methods argue that these approaches foster deeper conceptual understanding and mathematical reasoning. Critics, however, highlight the limited empirical support for inquiry-based methods, particularly for students who struggle with mathematics.

This tension remains visible in current professional guidance. Powell et al. (2025) reported that a recent joint position statement on mathematics instruction for students with disabilities, issued by two major professional organizations, cited only 34 sources, approximately half of which were organizational documents rather than peer-reviewed research, and only four of which addressed both mathematics and disability. This limited engagement with the disability-specific mathematics intervention literature is notable given the extensive experimental research base on mathematics instruction for students with disabilities and students with or at risk for mathematics difficulties. It also reflects a broader structural divide between mathematics education and special education that Cohen et al. (2025) documented empirically, finding that researchers in the two fields often operate in relative isolation, with implications for teacher preparation and student outcomes. For teacher preparation programs serving students with disabilities, this divide matters because effective preparation must attend to both the empirical intervention literature and to the contexts in which students are identified, served, and taught.

Witzel et al. (2024) similarly observed that the debate has resurfaced, with some scholars labeling explicit instruction as "dehumanizing" or "ableist," despite a lack of supporting student outcome data and substantial evidence demonstrating the effectiveness of explicit instruction for students with mathematics difficulties, particularly those with disabilities. Notably, the distinction between constructivism as a learning theory and as an instructional approach is important: Mercer et al. (1994) found that effective constructivist instruction in practice often incorporates features of explicit instruction, such as modeling, scaffolding, guided dialogue, and corrective feedback, indicating that the dichotomy is largely artificial.

Empirical literature has largely resolved the instructional debate in favor of structured, explicit approaches for students with mathematics difficulties, while also affirming that conceptual understanding and procedural fluency are mutually reinforcing objectives (Rittle-Johnson et al., 2015). Morgan et al. (2015) investigated instructional practices for first-grade students and reported that teacher-directed approaches yielded stronger gains for students with mathematics difficulties, with outcomes comparable to or exceeding those of student-centered approaches, even among students performing at grade level. Similarly, a large-scale randomized study of elementary mathematics curricula found that achievement gains were associated with programs that reflected explicit instruction over multiple consecutive years (Agodini & Harris, 2016).

Meta-analyses conducted over several decades, including Swanson and Hoskyn (1998), Kroesbergen and Van Luit (2003), Gersten et al. (2009b), and Stockard et al. (2018), have documented robust, replicable effects of explicit and direct instruction across content areas and student populations. The scope of this evidence is considerable: Powell et al. (2025) identified 36 research syntheses encompassing 836 experimental studies specifically focused on mathematics instruction for students with disabilities, highlighting that the empirical case for structured, explicit approaches is both broad and unambiguous. The National Mathematics

Advisory Panel (2008) synthesized this body of evidence and recommended that students experiencing mathematics difficulties receive systematic, explicit instruction, noting its documented track record across both computational and word-problem outcomes. These findings do not suggest that explicit instruction is the only effective approach; rather, they establish it as a necessary and well-supported component of effective mathematics instruction and intervention for students requiring specially designed instruction to access grade-level content.

1.3. Landmark Evidence Syntheses in Mathematics Instruction

Several landmark syntheses of evidence have shaped mathematics instruction and intervention in the United States, establishing shared frameworks for effective teaching across both general and special education contexts. *Adding It Up* (Kilpatrick et al., 2001), published by the National Research Council, introduced a five-strand framework (i.e., the math rope) for mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. This framework demonstrates that mathematical competence is multidimensional and that effective instruction must integrate all five strands rather than emphasizing a single component. The National Mathematics Advisory Panel (2008) subsequently synthesized a broad body of experimental and quasi-experimental research and issued recommendations on curriculum, instruction, teacher preparation, and assessment. For students with mathematics disability or difficulties, the Panel identified explicit instruction as a well-supported approach and emphasized foundational skills, particularly fluency with whole numbers, fractions, and the foundations of algebra, as prerequisites for long-term mathematics achievement (Star et al., 2015). More recently, Fuchs et al. (2021) published an IES Practice Guide summarizing empirically derived instructional practices to assist students struggling with mathematics in the elementary grades, translating intervention research into five empirically validated recommendations with implementation guidance. Collectively, these syntheses provide a convergent body of evidence and form the empirical foundation for current frameworks for mathematics instruction and intervention, including the present Innovation Configuration.

1.4. IES Practice Guide Recommendations for Mathematics Instruction

The Institute of Education Sciences (IES), part of the U.S. Department of Education, has developed a series of practice guides through the What Works Clearinghouse to translate research on mathematics instruction into actionable, empirically validated recommendations for practitioners. Each guide targets a specific domain or population and is developed by panels of researchers and practitioners who systematically evaluate the strength of available evidence for each recommendation. Notable guides include *Teaching Math to Young Children* (Frye et al., 2013), which addressed foundational number and operations concepts in early childhood; *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (Siegler et al., 2010), which synthesized evidence on persistent challenges in fractions for students with and without disabilities; *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (Star et al., 2015); and *Assisting Students Struggling with Mathematics: Response to Intervention for Elementary and Middle Schools* (Gersten et al., 2009a), which focused on tiered instructional support for students requiring intervention. Collectively, these guides demonstrated a sustained federal commitment to bridging the research-to-practice gap in mathematics education. Fuchs et al. (2021) further advanced this effort by producing the most recent IES practice guide specifically for elementary students struggling with mathematics, synthesizing intervention research from these guides and the broader literature into an integrated set of empirically validated recommendations for instructing students with mathematics disability or difficulties.

Several instructional recommendations recur consistently across these practice guides, indicating convergent evidence rather than the preferences of individual panels. Explicit and systematic instruction, characterized by direct teaching of concepts and procedures with clear modeling, guided practice, and corrective feedback, is recommended for students who struggle with mathematics (Frye et al., 2013; Fuchs et al., 2021; Gersten et al., 2009a). The use of visual and concrete representations to support understanding of abstract mathematical concepts is also recommended across guides spanning early childhood through algebra (Frye et al., 2013; Fuchs et al., 2021; Gersten et al., 2009a; Star et al., 2015). Number lines are emphasized as tools for developing understanding of magnitude across whole numbers, fractions, and integers (Fuchs et al., 2021; Siegler et al., 2010). Word-problem solving, including instruction in problem structure and the use of visual schemas, is identified as a high-priority skill area requiring dedicated, systematic, and explicit instruction (Fuchs et al., 2021; Woodward et al., 2012). Progress monitoring through frequent, curriculum-aligned assessments is recommended to inform instructional decisions and ensure students respond to intervention (Gersten et al., 2009a). The convergence of these recommendations across independently developed guides strengthens their evidentiary basis and provides a coherent framework for the empirically validated mathematics instruction described in this Innovation Configuration.

1.5. Federal Legislative Mandates Governing Mathematics Instruction and Intervention

Federal legislation has played a central role in shaping the expectations, structures, and accountability mechanisms that govern mathematics instruction for students with disabilities in the United States. The Individuals with Disabilities Education Act (IDEA, 2004) establishes the legal foundation for special education by guaranteeing eligible students a free appropriate public education (FAPE) delivered through specially designed instruction tailored to each student's unique needs (34 C.F.R. §300.39[b][3]). The landmark Supreme Court ruling in *Endrew F. v. Douglas County School District* (2017) clarified that FAPE requires an educational program reasonably calculated to enable meaningful progress appropriate to a student's individual circumstances, raising the bar beyond mere minimal advancement.

Despite this legal framework, Kouo et al. (2024) reported that state-level guidance on determining the type and amount of special education services varies considerably, with some states relying on formulaic rather than individualized approaches to IEP service determination, a practice inconsistent with IDEA's intent. As of 2020, only 66% of students with IEPs were educated in general education settings for 80% or more of the school day, with only modest growth over the prior decade (Kouo et al., 2024). The No Child Left Behind Act (NCLB, 2001) introduced federal accountability requirements mandating that all students, including those with disabilities, demonstrate proficiency in mathematics, significantly elevating expectations for students who had historically been excluded from such accountability systems. The Every Student Succeeds Act (ESSA, 2015), which replaced NCLB, maintained this accountability emphasis while establishing a four-tier evidence framework, strong, moderate, promising, and demonstrating a rationale, that requires federally funded educational programs and interventions to be supported by research evidence commensurate with the tier designated (ESSA, 2015; Witzel et al., 2026). This framework has direct implications for mathematics instruction and intervention for students with disabilities, as it institutionalizes the expectation that schools and districts select and implement practices with demonstrated empirical support, effectively codifying the use of EVPs as a federal requirement rather than a professional preference.

1.6. Nationwide Curricular Initiatives and Standards-Based Reform

Nationwide initiatives have substantially influenced the content, expectations, and delivery of mathematics instruction in the United States, establishing a unified policy context for special education mathematics instruction and intervention. These initiatives encompass voluntary state-led standards movements as well as federally promoted accountability and assessment systems, each shaping expectations for student learning, methods of measuring progress, and definitions of adequate instruction for both students with and without disabilities. Two standards initiatives have most substantially influenced mathematics curriculum and instruction over the past three decades. First, the National Council of Teachers of Mathematics released its *Principles and Standards for School Mathematics* in 2000 (NCTM, 2000), building on its 1989 standards by emphasizing conceptual understanding, mathematical reasoning, and problem solving as central goals of mathematics education. Second, the *Common Core State Standards for Mathematics* (CCSS-M), developed through a state-led initiative coordinated by the National Governors Association and the Council of Chief State School Officers (NGA & CCSSO, 2010) and released in 2010, established consistent academic expectations in mathematics across states, organizing content around a coherent progression from kindergarten through grade 12 while reinstating procedural fluency as a foundational goal alongside conceptual understanding and application.

For students with mathematics difficulties, both initiatives raised expectations by establishing grade-level content standards as the target for all students, including those receiving SDI, while leaving instructional decisions, including accommodations, modifications, and the selection of empirically validated practices, to educators and IEP teams (Powell et al., 2013). Although the CCSS-M became politically contentious and several states formally withdrew, most adopted functionally equivalent standards under state-specific names, preserving the essential content framework (Powell et al., 2013). Both initiatives also introduced standards for mathematical practice and process that describe the reasoning, communication, and problem-solving competencies students should develop alongside content knowledge, expectations that SDI must address in an integrated manner (Witzel et al., 2026).

Empirical research has investigated whether standards-based reform eras have influenced the effectiveness of mathematics interventions for students with difficulties and disabilities, with publication era consistently emerging as a significant predictor of effect size across multiple meta-analyses. However, the direction and magnitude of this moderating effect have varied across studies. Lein et al. (2020) reported that studies published between 2000 and 2009, corresponding to the NCTM era, yielded higher effects than those published from 2010 to 2019. In contrast, Myers et al. (2022b) found that studies conducted during the CCSS-M era (2010–2021) produced larger effects than those from the NCTM era, while observing no significant difference between pre-NCTM and NCTM-era studies. Most recently, Douglas et al. (2026) determined that studies conducted before the NCTM standards produced larger effect sizes than those from either the NCTM or CCSS-M eras. However, this finding was based on a limited number of pre-NCTM studies and should be interpreted with caution. Across all three meta-analyses, the small sample of pre-NCTM studies restricts confidence in comparisons involving that era. These inconsistencies likely result from differences in statistical design, inclusion criteria, and sample composition across meta-analyses, rather than indicating a definitive directional relationship. Nevertheless, the consistent identification of publication era as a significant moderator suggests that large-scale policy shifts in mathematics education, particularly changes in curricular emphasis on fluency, reasoning, and problem solving, have substantially shaped both the nature of mathematics interventions and their measured effectiveness over time. This underscores the importance of considering policy context when interpreting the intervention research base.

1.7. Professional Organization Standards for Mathematics Instruction

Professional organizations and national centers have contributed significantly to mathematics instruction and intervention for students with disabilities by developing standards, frameworks, and position statements that guide practitioner preparation, professional development, and instructional decision-making. Unlike federal legislation, these organizational standards and frameworks carry advisory rather than legal authority. However, they exert considerable influence on teacher education programs, certification requirements, and the broader professional culture of special education and mathematics education. The CEEDAR Center (Collaboration for Effective Educator Development, Accountability, and Reform), a national center funded by the Office of Special Education Programs to improve the preparation and professional development of teachers who serve students with disabilities, has played a central role in translating research on effective instruction into practitioner-facing frameworks and resources, including the Innovation Configuration format used in the present document.

The Council for Exceptional Children (CEC), in partnership with the CEEDAR Center, has been particularly influential in developing High Leverage Practices (HLPs) for special education, a set of 22 foundational teaching practices identified as essential for improving outcomes for students with disabilities (McLeskey et al., 2017). The HLPs are organized across four domains: collaboration, assessment, social-emotional and behavioral support, and instruction, and are intended to serve as the core of special educator preparation and practice. CEC subsequently released a second edition of the HLPs and the CEEDAR Center, refining and updating the framework to reflect evolving evidence and professional consensus (Aceves & Kennedy, 2024). Recent scholarship has further examined the practical application of the instructional HLPs in inclusive settings, identifying their relevance across general education, special education, and intervention contexts as a unified framework for meeting the needs of diverse learners (McKeithan et al., 2025). Several HLPs are directly relevant to mathematics instruction, including those addressing explicit instruction, flexible grouping, use of assistive technology, and intensive intervention. However, Nelson et al. (2021, 2022) examined the empirical basis of the HLPs and found that the level of research support varied considerably across practices, with some HLPs resting on a stronger evidentiary foundation than others, highlighting the need for continued research to validate and refine the framework.

The National Council of Teachers of Mathematics (NCTM) has similarly shaped mathematics instruction through its standards documents and position statements, including its advocacy for equity-oriented, conceptually rich instruction accessible to all learners. However, the influence of professional organization guidance is not uniformly grounded in rigorous empirical research specific to students with disabilities. Powell et al. (2025) examined a recent joint position statement on mathematics instruction for students with disabilities issued by NCTM and CEC and found that it drew on a narrow and largely non-experimental literature base, with only four of its 34 cited sources addressing both mathematics and disability. Gersten (2025) similarly critiqued the position statement for failing to engage substantively with the large body of high-quality intervention research accumulated over the preceding two decades, noting that five of the six recommendations from the most rigorous IES practice guide on elementary mathematics intervention went unacknowledged. Witzel et al. (2024) similarly documented that some professional discourse has characterized empirically validated instructional approaches as ideologically problematic despite robust empirical support for their effectiveness with students who experience mathematics difficulties. Taken together, these critiques underscore the importance of ensuring that professional organization standards are grounded in and accountable to the experimental research base, a principle that animates the present Innovation Configuration.

1.8. Influence of Public Media in Mathematics Policy and Practice

Public media, social media platforms such as Facebook, and advocacy-oriented outlets have increasingly shaped public understanding of mathematics education. These influences affect policy decisions, parental expectations, and practitioner choices, often operating outside the peer-reviewed research process. For example, the contentious debate surrounding the Common Core State Standards (NGA & CCSSO, 2010) was amplified by social media campaigns, parent advocacy networks, and news media coverage. These channels frequently reduced complex instructional and policy issues to simplified narratives, contributing to state-level political decisions that were not always grounded in evidence on student outcomes. More recently, the *Science of Reading* movement, propelled by investigative journalism, podcasts, and public advocacy prior to formal policy adoption, has illustrated both the potential and the limitations of public media as a vehicle for education reform. When public advocacy aligns with the peer-reviewed research base, as seen in the Science of Reading movement, media influence can accelerate the adoption of empirically validated practices at scale. Conversely, when advocacy diverges from research, it can reinforce misconceptions and generate political resistance to reform.

A comparable movement is emerging in mathematics education. *The Science of Math* (www.thescienceofmath.com) is a publicly accessible platform dedicated to using objective evidence on how students learn mathematics to inform educational decisions and policy. Consistent with peer-reviewed research on mathematics instruction and intervention, the Science of Math promotes instructional approaches supported by rigorous evidence, including findings from special education and psychology journals relevant to students with and without disabilities. The platform asserts that access to effective mathematics instruction is a fundamental educational right. Current proficiency data, which indicate that most U.S. students consistently do not meet minimum proficiency benchmarks, highlight the urgency of grounding mathematics instruction in research-supported practices (NCES, 2026). Although *the Science of Math* has not yet achieved the policy formalization seen in the *Science of Reading* movement, its emergence signals growing recognition among researchers, practitioners, and advocates. The persistent gap between empirically validated recommendations and classroom practice in mathematics requires sustained public attention alongside academic scholarship. This IC embodies this commitment by translating the peer-reviewed research base into actionable guidance for educator preparation programs and professional practice.

2.0. Foundations for Mathematics Teaching and Learning

Effective mathematics instruction for students with mathematics difficulties rests on a well-grounded understanding of how mathematics learning is structured, who struggles and why, what a comprehensive view of mathematical proficiency requires, and how learning unfolds over time. This section synthesizes five foundational dimensions of that understanding. It begins with the cumulative and hierarchical nature of mathematics and the documented trajectory from arithmetic to algebra (2.1) and then examines the cognitive, academic, and linguistic characteristics of learners with mathematics difficulties or disabilities (2.2). It next considers the five-strand model of mathematical proficiency established in Adding It Up (2.3) and theoretical and conceptual frameworks for how students learn mathematics (2.4) and concludes with the instructional hierarchy as a model for sequencing and intensifying instruction (2.5). These frameworks collectively inform the empirically validated practices described in subsequent sections of this Innovation Configuration.

2.1. The Cumulative Nature of Mathematics: From Arithmetic to Algebra

Mathematics learning builds sequentially, with later achievement depending on earlier conceptual foundations (National Council of Teachers of Mathematics, 2000). The trajectory begins in early childhood. Longitudinal evidence shows that mathematics knowledge at kindergarten entry is the single strongest predictor of later academic achievement across all subject areas (Duncan et al., 2007). Student learning in the elementary grades sets the trajectory for their future success in mathematics. During this period, students develop core understandings of number sense and operations that support subsequent learning (National Council of Teachers of Mathematics, 2000; Witzel, 2016). By Grade 5, mastery of fractions and division becomes particularly consequential, functioning as a unique predictor of success in algebra and later high school mathematics (Siegler et al., 2012). As students move into secondary school, mathematical competence continues to shape long-term outcomes, influencing college access and employment prospects (Lee, 2012).

A critical juncture in this progression occurs at the transition from arithmetic to algebra, often referred to as the "*arithmetic to algebra gap*" (Pillay et al., 1998; Witzel, 2016). Algebra represents a pivotal turning point in students' academic trajectories and a strong predictor of college completion (Adelman, 2006; Witzel et al., 2003). Many students, particularly those with mathematics difficulties, struggle to make this transition because they lack foundational arithmetic concepts and reasoning skills that algebra requires (Ketterlin-Geller et al., 2007; Ketterlin-Geller & Chard, 2011; Star et al., 2015; Witzel, 2016). Research shows that the strongest predictors of algebra readiness include computational fluency with whole numbers and fractions, understanding of equivalence and the meaning of the equals sign, facility with proportional reasoning, and the ability to recognize and extend numerical patterns (Pillay et al., 1998; Siegler et al., 2012; Watts et al., 2014; Witzel, 2016). Students must develop an understanding of variables and constants, the ability to decompose and represent word problems algebraically, the ability to manipulate symbols, and an understanding of functional relationships to develop algebraic thinking (Hughes et al., 2014; Ketterlin-Geller & Chard, 2011; Witzel, 2016; Woodward et al., 2012).

Bridging the gap between arithmetic and algebra requires explicit attention to algebraic thinking throughout elementary instruction rather than treating algebra as a discrete secondary school subject (Hughes et al., 2014; Ketterlin-Geller & Chard, 2011; Powell et al., 2016; Witzel, 2016). Elementary teachers can support this transition in several ways. Emphasizing a relational understanding of arithmetic operations and teaching the equal sign as representing equivalence, rather than simply indicating "the answer," builds a crucial foundation for algebraic thinking. Introducing variables as placeholders for unknown quantities in word problems helps students begin to formalize their reasoning (Jitendra et al., 2015; Myers & Witzel, 2023; Witzel et al., 2022). Teachers can further develop this understanding by using function tables and input-output relationships to explore mathematical patterns. Finally, connecting fraction concepts explicitly to proportional reasoning and algebraic notation reinforces these foundational ideas and prepares students for more advanced topics (Ketterlin-Geller & Chard, 2011; Witzel, 2016). For students with mathematics difficulties, systematic attention to these foundational concepts during the elementary years, particularly Grades 3 through 5, is essential to prevent long-term mathematics failure (Witzel, 2016). When early weaknesses in number sense, computational fluency, and fractional reasoning remain unresolved, they create barriers to later academic and professional opportunities, as algebra serves as a gateway to advanced mathematics coursework and STEM careers (Adelman, 2006; National Mathematics Advisory Panel, 2008).

2.2. Characteristics of Learners with Mathematics Difficulties

Students with mathematics difficulties represent a heterogeneous population that includes two overlapping but distinct groups. The first group comprises students with specific learning disability in mathematics, often diagnosed as dyscalculia outside the United States (Devine et al., 2018; Dowker, 2024; Hughes et al., 2023). These students have neurologically based processing disorders that interfere with their ability to perform mathematical calculations, apply logical and algebraic reasoning, and retrieve information from working and long-term memory, and they receive special education services with IEPs containing mathematics-focused goals (Geary, 2013; Hughes et al., 2023; Individuals with Disabilities Education Act, [IDEA] 2004; Mazzocco, 2007). The second, larger group consists of students without formal disability labels who persistently score below established cutoff points on mathematics assessments, typically performing below the 25th, 30th, or 35th percentile (Nelson & Powell, 2018). These students display deficits in essential skills, such as computational fluency, language comprehension, and number sense, as well as in domain-general skills, such as working memory (Nelson & Powell, 2018). While identification criteria vary across studies, research shows that both groups demonstrate similar mathematical challenges and, most notably, both benefit from the same empirically validated instructional practices (Fuchs et al., 2017; Fuchs et al., 2021; Myers et al., 2021).

Students with mathematics difficulties face challenges across cognitive, academic, and linguistic domains that interfere with mathematical learning and performance (Kong & Swanson, 2019; Swanson et al., 2018). Cognitively, these students demonstrate limitations in working memory, which constrains their ability to hold and manipulate information during problem solving, as well as slower processing speeds that impede automaticity with basic facts and nonverbal reasoning difficulties that affect pattern recognition and spatial reasoning (Cirino et al., 2015; Geary, 2013; Peterson et al., 2017; Swanson et al., 2024; Raghubar et al., 2010; Willcutt et al., 2013). These cognitive processes are essential for coordinating multiple solution steps and monitoring strategies during problem solving, particularly in word problems that require integrating linguistic and mathematical information (Swanson et al., 2024). Additionally, short-term memory supports the passive storage of problem information, while working memory enables the active manipulation of this information through interactions with phonological and visuospatial systems (Swanson et al., 2024).

Academically, many students with mathematics difficulties struggle with foundational number sense, including magnitude representation, counting, comparison, and understanding of number systems, all critical precursors to algebra (Aunio et al., 2015; Ketterlin-Geller &

Chard, 2011; Slot et al., 2016; Toll & Van Luit, 2014). Mathematical language presents additional challenges, as students often struggle to interpret terms describing operations and relationships, further complicating problem-solving performance (Powell et al., 2022; Toll & Van Luit, 2014). Fraction concepts present particularly persistent challenges (Hallett et al., 2010; Shin & Bryant, 2015). Students often overgeneralize whole-number concepts to fractions, and a weak conceptual understanding of fractions directly predicts poor performance in computation, estimation, and word-problem solving (Bottge et al., 2014; Hecht et al., 2003; Hwang et al., 2019; Siegler et al., 2010).

Many students with mathematics difficulties also experience comorbid reading difficulties, which intensifies their challenges and creates a distinct profile of cognitive and academic characteristics (Powell et al., 2009). While students with mathematics difficulties alone tend to show weaknesses in nonverbal reasoning and processing speed, students with reading difficulties demonstrate challenges with verbal working memory and language comprehension (Cirino et al., 2015; Willcutt et al., 2013). Students with both mathematics and reading difficulties display co-occurring weaknesses across domain-specific mathematics skills (such as calculation and early numeracy), domain-specific reading skills (including vocabulary knowledge and reading comprehension), and domain-general cognitive processes (such as working memory and short-term memory) that are essential for coordinating information during problem solving (Swanson et al., 2013; Swanson et al., 2024).

Because word problem solving requires integrating reading comprehension, vocabulary knowledge, and mathematical reasoning, students with mathematics and reading difficulties consistently score lower on word problem tasks than peers with mathematics difficulties alone or reading difficulties alone, and intervention studies report significantly smaller effects for students with comorbid mathematics and reading difficulties compared to students with mathematics difficulties only (Cirino et al., 2015; Myers et al., 2022b; Powell et al., 2009; Willcutt et al., 2019; Zheng et al., 2013). Without targeted intervention during elementary grades, these unresolved difficulties create cumulative disadvantages, blocking access to algebra and limiting long-term academic and career trajectories (Adelman, 2006; National Mathematics Advisory Panel, 2008; Witzel, 2016).

2.3. Mathematical Proficiency: The Five-Strand Framework and Its Limitations

Adding It Up: Helping Children Learn Mathematics (National Research Council, 2001) presented a foundational framework for developing students' mathematical proficiency through five interconnected strands (i.e., the math rope): (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition. Conceptual understanding refers to comprehending mathematical concepts, operations, and relationships; for example, a student might explain that multiplication is repeated addition by representing 3×4 as adding 4 three times. Procedural fluency denotes the capacity to execute procedures flexibly, accurately, efficiently, and appropriately (National Research Council, 2001); for instance, a student correctly and quickly computes 56 divided by 7. Strategic competence encompasses the ability to formulate, represent, and solve mathematical problems, such as when a student uses an equation to find how many tiles are needed to cover a rectangular floor. Adaptive reasoning involves logical thought, reflection, explanation, and justification (National Research Council, 2001); for example, a student justifies why a particular solution strategy works using mathematical reasoning. Productive disposition is characterized by a consistent belief that mathematics is logical, valuable, and worth learning, as well as confidence in one's effort and ability (National Research Council, 2001). In the classroom, this might be seen when a student persists in solving a challenging problem and expresses a positive attitude toward learning math. The National Research Council (2001) asserts that these strands develop simultaneously, each reinforcing and supporting the others.

Although *Adding It Up* (2001) has significantly influenced mathematics education, concerns persist about its practical implementation, particularly for students who struggle in mathematics. Concurrently developing all five strands may overwhelm students with limited working memory or slower processing speeds. For instance, requiring adaptive reasoning before mastery of basic procedures can be counterproductive (Sweller, 2023). The framework remains largely theoretical and offers limited guidance for daily instructional practice, as highlighted in federal evaluations (National Mathematics Advisory Panel, 2008). Standardized assessments frequently emphasize procedural fluency rather than the broader set of competencies outlined in the framework. Drawing on a National Research Council review, Sussman and Wilson (2019) noted that achievement tests often prioritize computation and procedural skills, even when interventions target mathematical reasoning, conceptual understanding, and problem solving (p. 193). These tensions are especially consequential for students with mathematics difficulties. When foundational skills are fragile, it remains unclear how students are expected to engage meaningfully across all strands without explicit support for prerequisite knowledge (National Mathematics Advisory Panel, 2008).

In light of these challenges, instruction must be deliberately structured to support students with fragile foundations. Educators can mitigate cognitive load, systematically strengthen prerequisite skills, and implement empirically validated practices aligned to students' needs (Fuchs et al., 2017; Powell et al., 2017; Swanson et al., 2024; Sweller, 2023).

Research consistently highlights explicit instruction, including clear modeling of procedures and reasoning; scaffolded support that fades as competence develops; the strategic use of concrete and visual representations to ground abstract ideas; and frequent, targeted practice to consolidate both conceptual and procedural knowledge (Fuchs et al., 2021; Powell et al., 2025). Structured, guided problem solving, characterized by instructional scaffolding, further supports the transfer of understanding to application, particularly for learners requiring intensified support (Powell et al., 2025).

2.4. Theoretical and Conceptual Frameworks for Mathematics Teaching and Learning

Multiple theoretical perspectives explain how students learn mathematics, each emphasizing different cognitive and social mechanisms and leading to distinct instructional approaches. The primary frameworks include constructivism, which emphasizes active knowledge construction through experience and social interaction; cognitive science and information processing theories, which focus on how the brain encodes, stores, and retrieves mathematical knowledge; and sociocultural and embodied cognition perspectives, which highlight the roles of culture, language, and sensorimotor experience in mathematical thinking (National Research Council, 2001; 2005). Understanding these frameworks is particularly important when preparing teachers to work with students with mathematics difficulties, whose cognitive and learning characteristics make some theoretical approaches more effective than others in supporting mathematical learning (Montague et al., 2011; Powell et al., 2025).

Constructivist frameworks, rooted in the work of Piaget and Vygotsky, emphasize that students actively construct mathematical understanding through interaction with their environment and engagement in problem-solving experiences rather than passively receiving knowledge from teachers (Piaget, 1952; Vygotsky, 1978). Piaget's cognitive constructivism focuses on developmental phases and on how children build increasingly sophisticated mental structures to understand mathematical concepts (Piaget, 1952). Vygotsky's social constructivism emphasizes that learning occurs through social interaction within a learner's zone of proximal development, the space between what a student can do independently and what they can accomplish with support (Vygotsky, 1978). From this perspective, scaffolding, temporary support provided by teachers or peers that is gradually removed as competence develops, becomes central to instruction (Wood et al., 1976). Constructivist principles have influenced reform mathematics curricula that emphasize inquiry-based learning, collaborative problem-solving, and the development of conceptual understanding through exploration and discourse before introducing formal procedures (National Council of Teachers of Mathematics, 2000). These approaches position the teacher as a facilitator who creates rich problem-solving environments rather than as a direct transmitter of mathematical knowledge (Vygotsky, 1978).

Cognitive science and information-processing theories offer contrasting perspectives, focusing on how the brain processes, stores, and retrieves information during mathematical learning (Sweller & Chandler, 1991). These frameworks emphasize the critical role of working memory, the limited-capacity system responsible for temporarily holding and manipulating information during problem-solving, and long-term memory, where mathematical knowledge becomes stored and organized in schemas that can be retrieved efficiently (Baddeley, 2012; Sweller, 2024). Cognitive Load Theory (Sweller, 1988) posits that instruction must manage the demands placed on working memory. Specifically, teachers should minimize extraneous cognitive load, the unnecessary mental effort created by poor instructional design, while supporting relevant cognitive load, the productive mental effort devoted to building schemas (Sweller, 2024; Sweller et al., 2011). Within this framework, effective instruction includes explicit instruction in procedures and concepts, worked examples that demonstrate problem-solving processes, systematic practice to build automaticity with foundational skills, and careful sequencing to avoid overwhelming working memory (Kirschner, 2002; Rosenshine, 2012; Sweller et al., 2011). Automaticity, the ability to execute mathematical procedures and facts without conscious attention, becomes particularly important because it frees working memory resources for higher-level reasoning and problem-solving (Burns et al., 2010; Burns et al., 2015). Cognitive science perspectives have generated strong empirical support for explicit, systematic instruction as an effective approach for developing mathematical proficiency (Fuchs et al., 2021; Powell et al., 2023; Powell et al., 2025).

Additional theoretical perspectives contribute to understanding mathematical learning, though they have received less emphasis in special education research and practice. Sociocultural theory, an extension of Lev Vygotsky's work, positions mathematics as a set of culturally situated practices rather than a universal body of abstract knowledge (Campanilla et al., 2024; Steele, 2001). It emphasizes how students learn mathematics through participation in mathematical discourse communities and the use of cultural tools (Forman, 2013; Nasir & Cobb, 2007). Embodied cognition frameworks propose that mathematical thinking is grounded in sensorimotor experience, with some theorists arguing that concrete manipulatives and physical models activate motor and spatial systems that support the development of abstract mathematical concepts (Lakoff & Núñez, 2000). While these perspectives offer valuable insights into the social and physical dimensions of mathematical learning, they have been less fully developed into systematic instructional practices for students experiencing long-standing academic struggles, such as those with mathematics difficulties, compared to constructivist and cognitive science approaches (Krawec & Steinberg, 2019; Powell et al., 2025).

For students with mathematics difficulties, research demonstrates that instructional approaches aligned with cognitive science principles are substantially more effective than those emphasizing pure discovery or inquiry-based learning (Alfieri et al., 2011; Kirschner et al., 2006). Students with mathematics difficulties often show limitations in working memory capacity, slower processing speeds, and difficulties in building automaticity with foundational skills, precisely the cognitive functions emphasized in information-processing theories (Geary, 2013; Swanson et al., 2018). Constructivist approaches that require students to discover mathematical concepts through minimally guided problem-solving can overwhelm the limited working memory resources of students with mathematics difficulties, as they must simultaneously manage problem comprehension, strategy selection, calculation, and monitoring without the automated foundational skills that would reduce cognitive demands (Kirschner et al., 2006; Sweller et al., 2011).

Empirical evidence from meta-analyses has established explicit, systematic instruction as a highly effective approach for improving mathematics outcomes among students with mathematics difficulties, with particular benefits when instruction includes clear models of problem-solving procedures, guided practice with immediate feedback, and systematic review to support long-term retention (Chodura et al., 2015; Dennis et al., 2016; Gersten et al., 2009b; Kroesbergen & Van Luit, 2003; Jitendra et al., 2018; Stevens et al., 2018). However, the evidence base directly comparing explicit instruction to inquiry-based approaches for this specific population remains limited (Krawec & Steinberg, 2019). A small body of research suggests that inquiry-oriented approaches supplemented with explicit instruction and additional scaffolds, such as Enhanced Anchored Instruction, can yield significant but modest gains for students with disabilities (Bottge et al., 2007). However, these approaches are complex to implement and less efficient than systematic, explicit instruction alone, with limited research demonstrating consistent improvements in mathematics outcomes (Krawec & Steinberg, 2019). The broader cognitive science literature, while not focused exclusively on students with mathematics difficulties, indicates that minimally guided approaches are less effective for novice learners processing complex information (Alfieri et al., 2011; Kirschner et al., 2006).

Importantly, effective instruction for students with mathematics difficulties cannot focus exclusively on procedural fluency at the expense of conceptual understanding (Powell et al., 2025; Rittle-Johnson et al., 2015). Research shows that students with mathematics difficulties benefit from instruction that integrates explicit teaching of both concepts and procedures, uses visual representations to make abstract ideas concrete, and provides multiple opportunities to build connections between mathematical ideas (Fuchs et al., 2021; Powell et al, 2025; Witzel et al., 2003). Effective instruction applies cognitive science principles, systematic, explicit instruction, attention to cognitive load, to develop all five strands of mathematical proficiency described in Adding It Up (Fuchs et al., 2021; National Research Council, 2001; Powell et al., 2025) recognizing that students with mathematics difficulties require more intensive, carefully sequenced instruction to develop the integrated understanding that both constructivist and cognitive science perspectives value (Fuchs et al., 2017; Powell et al., 2025).

2.5. The Instructional Hierarchy as a Framework for Teaching Mathematics

The instructional hierarchy (Haring & Eaton, 1978) describes four phases of learning content. While our focus is on mathematics, these phases are important for learning any content, both inside and outside school settings. The four phases of the instructional hierarchy include acquisition, proficiency, generalization, and adaptation. The first phase is the acquisition phase, in which students learn a new skill or re-learn one. The assumption in this phase is that students have limited knowledge of the skill. The focus during the acquisition phase is on accuracy and on designing instruction to help students become accurate in a skill (Myers et al., 2021). When students are learning a new skill, explicit instruction is essential for them to acquire knowledge about that skill. Lots of interactive teacher modeling with student practice opportunities allow students to develop accuracy with a mathematical skill. Feedback to students as they practice is paramount (Duhon et al., 2015). When a teacher determines that students are fairly accurate (e.g., 80% accuracy on problems), then they can consider moving to the next phase of the instructional hierarchy.

The second phase of the instructional hierarchy is the proficiency phase. In this phase, the focus remains on accuracy, but efficiency also becomes important. Students should be able to perform a mathematical skill accurately and easily. Intentionally designed practice opportunities that encourage students to build efficiency are integral to the proficiency phase. Some students need more practice to improve fluency, while others need less (Burns et al., 2015; Douglas et al., 2026). It is often in discussions about efficiency that timed assessments come up. With any assessment, untimed conditions allow for determining student accuracy (i.e., how many problems a student answers correctly). Timed conditions allow for determining a student's efficiency (i.e., how many problems a student can solve accurately within a finite time). Recent evidence does not support the assumption that timed assessment conditions necessarily increase mathematics anxiety. For example, Maki et al. (2024) found no significant differences in students' reported mathematics anxiety between timed and untimed conditions. Teachers should therefore make decisions about the use of timed and untimed assessments based on instructional goals and students' phase of learning (e.g., acquisition versus proficiency).

The third phase of the hierarchy is called generalization. In this phase, students practice when to apply their skills to solve a specific math problem and when not to. For example, if students worked on addition with the partial sums strategy during the acquisition and proficiency phases of the instructional hierarchy, generalization involves knowing when to use the partial sums strategy (e.g., for addition computation problems) and when to use other strategies (e.g., for subtraction or multiplication computation problems). This sounds easy, but sometimes students struggle to know when to use a specific skill because they practiced it in isolation, without discrimination, across different types of mathematical problems. Therefore, to facilitate generalization, teachers need to consider a range of practice opportunities for students. Within a single lesson, students should participate in interleaved practice. This practice presents students with different types of problems (e.g., a mix of addition and subtraction), giving them the opportunity to practice when to add and when to subtract (Hughes & Riccomini, 2019). Across lessons, students should receive opportunities for distributed practice. This ensures that students get regular, intentional practice on skills they have learned in the past. This type of practice helps students continually practice previously learned skills so that they do not forget that skill knowledge.

The final phase of the instructional hierarchy focuses on adaptation. This is primarily focused on transfer—that is, taking a skill and using that knowledge to transfer and generalize it to unfamiliar or novel situations. Much of adaptation occurs through problem-solving, although problem-solving requires explicit instruction during acquisition and fluency practice during proficiency.

3.0. Fundamentals of Mathematics (Grades K-2)

Grades K-2 lay the mathematical foundations on which all subsequent learning depends (Frye et al., 2013). The standards addressed during these grades span five interconnected domains: (a) counting and cardinality, (b) operations and algebraic thinking, (c) number and operations in base 10, (d) measurement and data, and (e) geometry. Although these domains are treated separately in standards documents, they develop concurrently and reinforce one another. Mastery of early counting and cardinality concepts, for example, underpins the development of number sense and the subsequent operations. For students who struggle in mathematics, deficits in these foundational domains during the primary grades often predict persistent difficulties in later years (Aunio et al., 2015; Ketterlin-Geller & Chard, 2011). Teacher preparation programs must, therefore, ensure that candidates possess deep knowledge of K-2 mathematics content and understand how these early standards set the trajectory for the progression from arithmetic to algebra.

3.1. Counting and Cardinality

Counting and cardinality are kindergarten standards in which students use stable order and one-to-one correspondence to determine the number of objects in a set (i.e., cardinality; Frye et al., 2013). This set of standards also involves comparing two numbers to determine whether one is greater than, less than, or equal to the other. Skill with counting and cardinality is important for later success with the operations and beyond.

3.2. Operations and Algebraic Thinking

In kindergarten, grade 1, and grade 2, a set of standards primarily focused on addition and subtraction is organized under the Operations and Algebraic Thinking strand. Within this domain, a central focus is interpreting addition as both putting together and adding on, and interpreting subtraction as taking away and comparing quantities to determine a difference. In kindergarten, students are expected to represent addition and subtraction within 10 and develop fluency within 5 (Frye et al., 2013). In grade 1, students should be able to add and subtract within 20, with a particular focus on word problems; fluency with math facts within 20 is expected. Students should learn about the commutative and associative properties of addition and be able to work with equations in which the unknown is the addend. For second-grade students, standards outline the use of addition and subtraction within 100 for word problems; students are expected to know all 100 addition facts by memory. Odd and even numbers are emphasized in grade 2, along with the precursors of multiplication (e.g., arranging objects into rows and columns) and skip counting with addition.

3.3. Number and Operations in Base 10

Students' knowledge of number and operations in base 10 is addressed in the kindergarten, grade 1, and grade 2 standards. In kindergarten, the focus is on composing and decomposing numbers from 11 to 19 into tens and ones. In grade 1, students are expected to count to 120 and represent all two-digit numbers. Addition computation (i.e., addition in steps) is taught with a two-digit addend and a one-digit addend, as well as addition and subtraction with 10 more or 10 less. In Grade 2, students expand their place-value knowledge to three-digit numbers, with a focus on the hundreds place. Students should be able to work with numbers up to 1,000 and compare numbers based on place value. Second-grade students add and subtract within 1,000 using different strategies, with an emphasis on hundreds, tens, and ones.

3.4. Measurement and Data

In kindergarten, the measurement and data strand focuses on measuring length and weight using standard and nonstandard units. Kindergarteners are also expected to sort objects and classify them by category. For first-grade students, measurement revolves around ordering objects by length and using iteration when measuring. Students are asked to tell time to the hour and half-hour increments. Data analysis is a focus in grade 1, in which students should be able to represent data and answer questions about more and less across categories. In grade 2, students use measuring tools to measure in standard units, often in both customary (i.e., inches, feet) and metric (i.e., centimeters) units. Students apply their addition and subtraction knowledge to solve word problems about length. Students are also expected to use their knowledge of the number line to add and subtract. Second-grade students continue to work on time and learn about a.m. and p.m. They also focus on word problems with money. For Grade 2 data, students generate picture or bar graphs to represent data and solve word problems using information presented in bar graphs.

3.5. Geometry

In kindergarten, geometry standards focus on identifying two-dimensional shapes such as rectangles, triangles, and circles, as well as three-dimensional figures such as cubes, cones, and spheres. Students are expected to compare and analyze shapes based on attributes. Grade 1 geometry standards expect students to compare two-dimensional shapes and three-dimensional figures. Students begin to partition (i.e., divide) shapes into equal parts, a precursor to working with fractions. Grade 2 continues with standards about partitioning shapes into equal parts. Students also do more work with angles and other attributes for more complex two-dimensional shapes (e.g., hexagons) and three-dimensional figures.

4.0. Fundamentals of Mathematics (Grades 3-5)

Grades 3-5 mark a significant transition in the mathematical demands placed on students. While the primary grades build foundational number sense and early operations, the upper elementary years introduce multiplication, division, fractions, decimals, multi-step problem solving, and the precursors of algebraic reasoning. These content areas are not merely extensions of earlier learning; they represent qualitative shifts in mathematical complexity that require students to coordinate multiple representations, reason about relationships among quantities, and apply procedures flexibly across novel contexts (National Mathematics Advisory Panel, 2008; Siegler et al., 2010). For students with mathematics difficulties, the upper elementary years are often when performance gaps widen, and the consequences for long-term trajectories become pronounced (Witzel, 2016). The five domains addressed in this section, (a) operations and algebraic thinking, (b) number and operations in base 10, (c) number and operations with fractions, (d) measurement and data, and (e) geometry,, reflect the content knowledge that teachers must command to deliver appropriately intensive, specially designed instruction in Grades 3–5.

4.1. Operations and Algebraic Thinking

In the late elementary grades, the operations and algebraic thinking strand continues to include many standards. In grade 3, multiplication and division are formally introduced, and students are expected to use these operations with numbers up to 100 to solve basic facts and word problems. Work with unknown factors in multiplication and unknown divisors or dividends in division problems reflects early algebraic thinking. Students also begin solving more complex, multi-step word problems. In grade 4, solving multi-step word problems involving addition and subtraction remains an important focus, and students learn to determine factors and multiples of whole numbers. At this level, students are also expected to develop fluency with the 190 multiplication and division facts. By grade 5, instruction extends to working with numerical expressions and generating number patterns based on given rules.

4.2. Number and Operations in Base 10

As in grades K–2, number and operations in base 10 remains a focus of the standards in the late elementary grades. For third-grade students, expectations relate to rounding whole numbers to the nearest 10 or 100. Students work with multi-digit numbers to add and subtract within 1,000. Understanding products when multiplying by 10 (e.g., 6 times 10) is another focus of Grade 3 mathematics. In Grade 4, students expand their rounding skills to round to any place value. Students compare numbers based on place value. Beyond multi-digit addition and subtraction, students multiply a one-digit number by a four-digit number and divide a four-digit dividend by a one-digit divisor. Students work with remainders for division. Fifth-grade students are expected to understand place value in the base-ten system, with emphasis on powers of 10. Students should compare decimals to the thousandths and round decimals to any place value. Grade 5 students should be fluent in multi-digit multiplication and division with whole numbers. Students should be able to solve computation problems across operations to the hundredths place.

4.3. Number and Operations with Fractions

In grade 3, a new set of standards addressing number and operations with fractions is introduced. Students learn that a fraction is composed of equal parts of a whole, area, or set, and they learn how to represent fractions on a number line. They focus on fraction equivalence and comparing fractions. Grade 4 places even greater emphasis on fractions. Students work with equivalent fractions and compare fractions. They also think about the addition and subtraction of fractions, particularly with unit fractions (i.e., a fraction with a numerator of 1). Students solve word problems involving addition and subtraction, as well as multiplication of a fraction by a whole number. Grade 4 also introduces decimals. Students learn the relationship between fractions and decimals, and they compare decimals. In Grade 5, students add and subtract fractions with unlike denominators and solve word problems with these numbers. They also solve word problems involving multiplication, division, and fractions. Students focus on scaling.

4.4. Measurement and Data

Standards addressing measurement and data develop progressively across the late elementary grades. In grade 3, students are expected to tell time to the nearest minute and solve word problems involving time. They also work with liquid volume using customary and metric units, while data analysis focuses on interpreting scaled bar graphs. Instruction introduces the use of unit squares to calculate the area of two-dimensional shapes, and multiplication and addition are used to calculate perimeter and area. In Grade 4, students use customary and metric units to solve word problems across the four operations involving time, volume, mass, and money. They continue calculating perimeter and area and create line plots that include fractional values, using these representations to add and subtract fractions.

Students also learn about angles and how to measure them. By Grade 5, students convert among measurement units and continue working with line plots. Instruction also emphasizes determining the volume of cubes and other solid figures.

4.5. Geometry

Students have several geometry expectations in the later elementary grades. In grade 3, students focus on shared attributes of two-dimensional shapes and three-dimensional figures. They continue to partition shapes into equal parts. In grade 4, students focus on points, lines, line segments, rays, angles (e.g., right, acute, and obtuse), perpendicular lines, and parallel lines. Students also focus on lines of symmetry. In Grade 5, students work with a coordinate plane and plot ordered pairs. Students graph on the coordinate plane. For two-dimensional shapes, students group such shapes into categories based on their attributes.

5.0. Empirically Validated Practices for Teaching the Fundamentals of Mathematics

Decades of experimental research have identified a set of instructional practices that reliably improve mathematics outcomes for students with difficulties and disabilities when implemented with fidelity (Fuchs et al., 2021; Powell et al., 2025). This section synthesizes seven of those practices: systematic, explicit instruction (5.1); mathematical language instruction (5.2); representational approaches, including the concrete–representational–abstract framework (5.3); number line instruction (5.4); fluency-building (5.5); word-problem solving (5.6); the needs of dual-language learners (5.7), and (5.8) student expression in mathematics. Each practice is described in terms of its theoretical rationale, empirical evidence base, and core implementation features. While these practices are treated individually for clarity of presentation, they do not operate in isolation; effective mathematics instruction and intervention integrate them as a coherent bundle, with implementation intensity calibrated to the phase of learning and the individual needs of each student. Teacher preparation programs are responsible for ensuring that candidates understand each practice, can implement them with fidelity, and can combine them purposefully within the multi-tiered support structures described in Section 6.

5.1. Systematic, Explicit Instruction

Systematic, explicit instruction is a foundational approach to the design and delivery of mathematics intervention, characterized by intentional design features that build students' knowledge incrementally toward clearly defined learning outcomes (Engelmann & Carnine, 1982). This approach addresses the specific needs of students with mathematics difficulties through strategic sequencing of content and purposeful integration of instructional supports (Doabler & Fein, 2013; Fuchs et al., 2021; Powell et al., 2023). Systematic, explicit interventions typically employ a "bundle" of empirically validated practices that work collectively to support student learning, including sequential content design, regular review and integration of previously learned material, use of accessible numbers during initial concept introduction, visual and verbal supports, and immediate corrective feedback (Fuchs et al., 2021; Powell et al., 2025). The effectiveness of systematic, explicit instruction is well documented across multiple mathematics domains, with strong empirical evidence demonstrating positive effects on whole-number computation, rational-number knowledge, word-problem solving, and overall mathematics achievement among students experiencing mathematics difficulties (e.g., Dennis et al., 2026; Gersten et al., 2009b; Jitendra et al., 2018; Kong et al., 2021; Lein et al., 2020; Myers et al., 2021; Myers et al., 2022b; Stevens et al., 2018).

Implementing systematic, explicit instruction requires careful attention to both the overall intervention design and the specific instructional decisions made during daily lessons. Teachers must sequence instruction so that mathematical concepts and procedures build incrementally, with each lesson connecting to prior learning and advancing toward specific outcomes rather than focusing on isolated skills. Within this framework, teachers regularly review and integrate previously learned content to ensure students maintain their understanding and avoid overgeneralizing new concepts or procedures by systematically mixing problem types (Hughes et al., 2022; Rohrer et al., 2020). During initial instruction on new concepts and procedures, teachers should use simple, manageable numbers to help students focus on the underlying mathematical structure rather than on computational complexity. Throughout instruction, teachers provide visual and verbal supports, including strategic prompts, questions, gestures, and representations to help students attend to and remember connections between prior and new learning. These design features, incremental sequencing, integrated review, accessible numbers, and embedded supports, reflect the core principles of systematic, explicit instruction and are essential for students who require additional time and repeated exposure to develop a deep understanding of grade-level mathematics topics (Fuchs et al., 2021; Powell et al., 2025).

5.2. Mathematical Language

Teaching students to use clear, concise mathematical language is another empirically validated practice that supports students' mathematical knowledge across Grades K–5 (Fuchs et al., 2021; Lariviere et al., 2024). While mathematical language can refer to the symbols, numerals, terms, representations, and gestures of mathematics, one primary component of mathematical language is mathematical vocabulary. Student understanding of mathematical vocabulary is essential because vocabulary is important for success in all the ways students are expected to interact with mathematics, from listening to speaking to reading to writing. Across the early elementary grades, students may be expected to know approximately 200 distinct mathematical vocabulary terms, whereas in the later elementary grades, the number is approximately 500 (Powell et al., 2022). As demonstrated by Lin et al. (2021), knowledge of these mathematical vocabulary terms is significantly related to overall mathematics performance, particularly in word-problem solving. Unfortunately, many students struggle to recognize and define mathematical vocabulary terms (Powell et al., 2017), and these difficulties with mathematical vocabulary begin before kindergarten (Schmitt et al., 2019). Importantly, and related to this IC, targeted instruction in mathematical vocabulary can improve students' mathematics outcomes (Lariviere et al., 2024; Turan & DeSmedt, 2022).

To help students learn mathematics vocabulary, teachers must provide many opportunities for students to hear, use, read, and write formal mathematics vocabulary terms (Hughes et al., 2016). In addition to knowledge of formal terms, teachers must design instruction and practice to ensure students learn the precise definitions of each term (Powell et al., 2019). To develop proficiency with mathematics vocabulary, teachers should use a variety of activities, including word wall cards, vocabulary cards, graphic organizers, semantic maps (i.e., concept maps), and game-based activities (Lin & Powell, 2023; Patterson & Hicks, 2020; Stevens et al., 2023). As emphasized by Nelson and Kiss (2021), a routine for mathematics vocabulary could include teaching a student-friendly definition, encouraging students to use vocabulary in their discourse in the mathematics classroom or intervention, using representations to deepen conceptual and procedural knowledge of vocabulary, and creating a glossary of mathematics vocabulary terms.

5.3. Representations

Representations, such as physical, virtual, and visual models, serve as instructional bridges that transform abstract mathematical ideas into tangible, accessible forms for students experiencing difficulty with mathematics, thereby supporting the development of conceptual understanding and procedural fluency (Bouck et al., 2018; Powell et al., 2025; Satsangi et al., 2018; Witzel et al., 2003). Physical manipulatives, such as base-ten blocks, fraction tiles, and snap cubes, allow students to interact directly with mathematical concepts and procedures through hands-on exploration (Fuchs et al., 2021). These physical manipulatives have virtual equivalents presented on computer or tablet screens that serve similar instructional functions (Bouck & Flanagan, 2010; Park et al., 2022; Root et al., 2021). Visual models, such as pictures and drawings, further support learning by helping students organize information and perceive relationships among quantities (Fuchs et al., 2021). While core mathematics instruction often emphasizes abstract notation, students with mathematics difficulties require focused instruction using concrete (or virtual) and visual representations to build conceptual and procedural understanding before transitioning to symbolic work (Fuchs et al., 2021; Satsangi & Sigmon, 2024).

A substantial body of research supports the effectiveness of representation-based instruction for students with mathematics difficulties. Meta-analytic and experimental studies demonstrate positive effects across multiple mathematics outcomes, including whole-number computation, rational-number knowledge, fractions, and word-problem solving (Gersten et al., 2009b; Jitendra et al., 2021; Lein et al., 2020; Myers et al., 2021; Myers et al., 2023; Shin et al., 2021; Stevens et al., 2018). This instructional approach is commonly organized around a concrete–representational–abstract (CRA) framework. Evidence from randomized controlled trials and single-case design studies indicates that CRA-based instruction produces meaningful improvements across mathematics topics and diverse learner populations, including foundational number concepts and algebra (Bouck et al., 2018; Ebner et al., 2025;Witzel et al., 2003). Effective implementation of representations requires more than simply providing materials. Teachers must deliberately select representations that authentically model the mathematical concepts and procedures at hand, ensuring that physical relationships mirror numerical relationships, such as using place-value materials in which larger units are proportionally sized relative to smaller units (Fuchs et al., 2021). Instruction should also make explicit connections across levels of representation so that students understand how concrete and visual models map onto symbolic notation, rather than treating each level as a disconnected procedure.

5.4. Number Lines

Although number lines are often described as semi-concrete representations within the concrete–representational–abstract framework, their importance extends well beyond that classification. They provide a coherent structure for organizing the number system across grade levels. Because number lines preserve linear magnitude relationships, they allow students to situate whole numbers, integers, fractions, decimals, and early algebraic ideas such as inequalities within a single continuous space (Fuchs et al., 2021; Rodrigues et al., 2024). For students with mathematics difficulties, this continuity is critical. Rather than encountering each new topic as a separate symbolic system, students can anchor emerging concepts to an existing magnitude framework (Gersib et al., 2023).

Number lines strengthen students' understanding of relative magnitude by emphasizing distance from zero rather than isolated numerals. With repeated use, students begin to internalize this spatial structure, supporting the development of mental number lines that underlie estimation, comparison, and proportional reasoning. In this way, number lines function as more than visual supports; they scaffold problem representation and help students coordinate operations with movement along a measurable scale (Fuchs et al., 2021; Gersib et al., 2023). This representational clarity becomes increasingly important as students learn rational numbers, where precision depends on recognizing density and relative size rather than relying on whole-number intuitions (Dyson et al., 2020; Gersib et al., 2023; Mazzocco et al., 2013; Rojo et al., 2023). Effective instruction begins with embodied experiences, such as moving along number paths or using tactile measurement models, while explicitly linking those experiences to symbolic notation and attending to conventions such as uniform spacing and bidirectional extension (Gonsalves & Krawec, 2014; Namkung & Fuchs, 2016).

A growing body of research supports the instructional value of number lines for students with mathematics difficulties. Practice guides and experimental studies indicate that systematic, explicit number line instruction improves magnitude understanding and strengthens rational number reasoning across grade levels (e.g., Barbieri et al., 2020; Fuchs et al., 2016; Fuchs et al., 2021; Hamdan & Gunderson, 2017; Jayanthi et al., 2021; Zhang et al., 2017). Developmental and longitudinal research further suggests that improvements in number line accuracy reflect increasingly precise spatial representations of symbolic magnitude, which are associated with broader growth in number system knowledge and spatial reasoning (Fuchs et al., 2021; Gunderson et al., 2012; LeFevre et al., 2013). Taken together, this evidence positions number lines not as supplementary visuals but as a fundamental representational tool within mathematics intervention.

5.5. Fluency

Fluency is another empirically validated practice in the teaching and learning of mathematics (Burns et al., 2010; Codding et al., 2011; Douglas et al., 2026; Kim et al., 2022). Our definition of fluency aligns with Haring and Eaton's (1978) instructional hierarchy, in which students develop both accuracy and efficiency in a mathematics skill. Fluency is essential across grade levels and mathematics content (Douglas et al., 2026). That is, students should develop fluency—accuracy and ease—with counting, comparing numbers, adding, subtracting, identifying place value, multiplying, dividing, showing fractions, identifying decimals, interpreting graphs, identifying shapes, measuring, algebra, and more. Fluency only develops through intentionally designed practice opportunities with feedback (Codding et al., 2019; Schutte et al., 2015), and teachers should incorporate fluency-building activities into their mathematics instruction or intervention daily. Research indicates that students who experience difficulty with mathematics likely need many more practice opportunities to develop fluency than students without mathematics difficulty (Burns et al., 2015).

5.6. Word Problem Solving

Instruction in solving word problems is another important component of daily mathematics instruction or intervention. Why? All students in the United States demonstrate their mathematics knowledge on standardized and state-level tests by setting up and solving mathematics word problems. Within Haring and Eaton's (1978) instructional hierarchy, word-problem solving falls within the adaptation phase, as students transfer their learned mathematical knowledge to unfamiliar situations and problems. However, Word-Problem Solving needs to be supported within the acquisition (i.e., demonstrating how to approach word problems), proficiency (i.e., demonstrating accuracy and ease with solving word problems), and generalization (i.e., knowing how to solve different types of word problems) phases of the hierarchy. Students with a disability or difficulty often demonstrate lower performance on word problems than students without difficulty (Powell et al., 2015).

Empirically validated practices to support word-problem solving include the use of a meta-cognitive strategy (Freeman-Green et al., 2015; Montague et al., 2011), a focus on the concepts within word problems (i.e., schemas; Kong et al., 2020; Lien et al., 2020), use of graphic organizers (Jitendra et al., 2017; Root, Ingelin, et al., 2021), and use of representations (Browder et al., 2018). For students with mathematics difficulties, simultaneous implementation of multiple strategies, combining metacognitive, schema-based, and representational approaches, is often necessary to provide the intensive support required for successful word-problem solving (Lein et al., 2020; Myers & Witzel, 2023). However, the effectiveness of word-problem interventions is not uniform across all students with mathematics difficulties. Results of an umbrella review of 11 meta-analyses (Myers et al., 2024) identified four intervention and participant characteristics that consistently moderated word-problem outcomes: intervention model, number of treatment sessions, group size, and academic risk area. These findings suggest that when standard word-problem instruction is insufficient, teachers and interventionists should consider these variables when customizing and intensifying support—for example, by reducing group size, increasing the number of sessions, or selecting an intervention model aligned with a student's specific academic risk profile.

5.7. Needs of Dual Language Learners

English learners, defined as students whose primary home language is not English and who require additional support to access academic instruction delivered in English, encounter unique challenges in mathematics classrooms. Spanish-speaking students represent the majority of the EL population in U.S. schools and are disproportionately represented among students with persistent underperformance in mathematics (Lei et al., 2020). For ELs experiencing difficulty in mathematics, solving word problems is especially challenging because it requires managing both the linguistic complexity of the problem text and the mathematical reasoning required for the solution. These dual demands are particularly taxing for students whose academic language proficiency is still developing (Driver & Powell, 2017; Kong & Swanson, 2019). Low mathematics performance among English learners often reflects a misalignment between instructional design and students' linguistic needs, rather than a lack of mathematical understanding (Kong & Orosco, 2016; Orosco et al., 2013).

Effective instruction for English learners with mathematics difficulties attends to both the mathematical and linguistic dimensions of learning. Before introducing problem-solving tasks, teachers should explicitly teach vocabulary and relational language embedded in word problems (Kong & Swanson, 2019; Lei et al., 2020). Problems should be constructed or modified to reduce unnecessary linguistic load without diminishing mathematical demand, and visual and concrete representations should be used to give students access to problem structure when language constraints comprehension (Driver & Powell, 2017; Orosco, 2014). Centering students' prior knowledge, cultural experiences, and linguistic resources as assets in the mathematics classroom, can strengthen motivation and engagement when integrated into word-problem contexts (Gay, 2002; Orosco et al., 2013). Lei et al. (2020) conducted a meta-analysis of single-case intervention research targeting word-problem solving for English learners with learning disabilities and mathematics difficulties, finding a moderate overall effect. Notably, interventions that focused on mathematical problem-solving yielded stronger outcomes than those primarily oriented toward reading comprehension, suggesting that linguistic scaffolds should support rather than displace focused mathematics instruction.

Reading comprehension, vocabulary, calculation skills, and working memory all influence word-problem-solving performance for English learners with mathematics difficulties, with their relative importance varying among learners (Kong & Swanson, 2019; Swanson et al., 2020; Swanson et al., 2024). When possible, collecting assessment and progress-monitoring data in both students' home language and English provides a more comprehensive understanding of their mathematical knowledge and language development. These data should inform the planning of core instruction and supplemental interventions.

5.8. Student Expression in Mathematics: Verbalizations and Writing

Two instructional practices that receive less attention in mathematics intervention research than explicit instruction, worked examples, representations, and practice opportunities are student verbalizations of mathematical thinking and writing. Although these practices differ in form, both require students to generate and communicate mathematical reasoning. For students with mathematics difficulties, verbal and written expression can provide opportunities to clarify conceptual understanding, reveal errors in reasoning, and connect procedures with underlying mathematical ideas.

Student verbalizations include two related practices: self-explanation, in which students explain their reasoning during problem solving, and peer explanation, in which students communicate mathematical thinking to classmates. In a meta-analysis of self-explanation in mathematics, Rittle-Johnson et al. (2017) found that prompts requiring students to explain their reasoning produced stronger learning outcomes than comparable opportunities for additional practice. Effects were observed across grade levels and mathematics content areas. Self-explanation appears most useful when students are prompted to explain why a procedure works, rather than merely describing the steps they completed, because such prompts encourage students to connect procedural steps to conceptual meaning (Rittle-Johnson et al., 2017). Peer explanation may provide an additional benefit by requiring students to organize their reasoning for an audience and use more precise mathematical language. Teachers can support both forms of verbalization through structured prompts, sentence frames, and discourse routines that help students explain their thinking while keeping the focus on mathematics.

Writing in mathematics represents a second form of student expression with empirical support. Graham et al. (2020) conducted a meta-analysis of 56 experiments examining the effects of writing on learning in science, social studies, and mathematics across grades 1 through 12 and found that writing-to-learn activities had positive effects on content achievement across subject areas and grade levels. Graham and Perin (2007) also documented the academic benefits of writing-to-learn strategies for adolescent learners across content areas. In mathematics, written expressions may include explanations of problem-solving processes, justifications of solution strategies, written comparisons of approaches, and reflective journal entries. For students with mathematics disabilities and difficulties, Hughes et al. (2020) found that explicit instruction in mathematical writing supported written reasoning among middle school students with learning disabilities. Riccomini et al. (2024) similarly reported gains in mathematical reasoning and written expression when fifth-grade students with specific learning disabilities received structured instruction in explaining their mathematical thinking through writing.

Effective implementation requires teachers to treat mathematical writing as an instructional practice that must be modeled, scaffolded, and reinforced. Students often need explicit models, structured templates, guided practice, and feedback on the mathematical accuracy, completeness, and clarity of their written explanations. Feedback should address writing conventions when appropriate, but the central focus should remain on the quality of mathematical reasoning. As with verbal explanation, writing activities should support focused mathematics instruction rather than replace time needed for direct instruction, guided practice, and problem solving.

6.0. Characteristics of Effective Assessment, Instruction, and Intervention in Mathematics

Effective mathematics instruction for students with mathematics difficulties does not occur in isolation; it is embedded within coherent organizational and assessment systems that ensure instruction is appropriately intense, individually targeted, and systematically evaluated. This section describes three interdependent structures that operationalize those conditions: multi-tiered systems of support (6.1), which provides the prevention and intervention framework through which empirically validated practices are delivered; internally consistent mathematics IEPs (6.2), which translate legal mandates into individualized instructional plans aligned with ambitious, measurable goals; and data-based individualization (6.3), which structures the ongoing use of student performance data to adapt and intensify instruction when progress is insufficient. Together, these systems ensure that students with mathematics disabilities and difficulties receive instruction that is not only evidence-based in content and method but also sufficiently responsive and individualized to produce meaningful academic progress.

6.1. Core Components of Effective MTSS Instructional Frameworks

Multi-Tiered Systems of Support (MTSS) provide the organizational framework through which empirically validated instructional practices, including systematic, explicit instruction and mathematical representations such as number lines, are delivered with increasing intensity to meet individual student needs (Fuchs & Fuchs, 2017; Gersten et al., 2009a; Jackson, 2021). MTSS frameworks generally comprise four interdependent components that work together to prevent and address academic difficulty: universal screening, a multi-level prevention system, progress monitoring, and data-driven decision-making (Center on Multi-Tiered Systems of Support, 2022; Fuchs & Fuchs, 2017; Sailor et al., 2021).

Universal screening identifies students at risk for academic difficulty through brief assessments administered to all students at regular intervals throughout the school year (Fuchs & Fuchs, 2017). Selecting an appropriate screener requires attention to diagnostic accuracy, including sensitivity, or the proportion of students correctly identified as at risk, and specificity, or the proportion of students correctly identified as not at risk (Gersten et al., 2012; Nelson et al., 2025). Schools must weigh the tradeoffs between these indices carefully. Higher sensitivity reduces the likelihood of missing students who need intervention, whereas higher specificity reduces the number of students incorrectly identified for services they may not need (VanDerHeyden & Burns, 2013). An additional consideration is whether to apply publisher-recommended cut scores or locally derived cut scores, as classification accuracy can vary meaningfully depending on the approach used, particularly in schools serving demographically distinct populations (Nelson et al., 2025). Curriculum-based measures in mathematics provide technically adequate and time-efficient screening tools for the primary grades, though schools must also consider the tradeoffs between brief single-skill measures and more comprehensive multiskill assessments when making screening decisions (Gersten et al., 2012; Nelson et al., 2025).

The multi-level prevention system organizes instruction across three commonly used tiers. Tier 1 consists of high-quality core instruction provided to all students, Tier 2 provides targeted small-group intervention for students demonstrating emerging difficulties, and Tier 3 provides intensive, individualized intervention for students requiring the most substantial support (Fuchs & Fuchs, 2017; Zhang et al., 2023). Intervention intensity across tiers is shaped by several malleable variables that educators can adjust in response to student data. These include group size, session frequency and duration, the specificity of the targeted skill, and motivational components designed to sustain engagement over extended intervention periods (Fuchs et al., 2017; Myers et al., 2024). Increasing dosage, particularly the frequency and number of sessions, is among the most consistently supported intensification strategies for students who demonstrate insufficient response to standard Tier 2 intervention (Douglas et al., 2026; Myers et al., 2022b).

Progress monitoring uses reliable, curriculum-aligned measures administered frequently to assess students' responsiveness to instruction, with data used to inform adjustments to intervention intensity, duration, or approach (Fuchs & Fuchs, 2017). Selecting an appropriate progress monitoring measure requires understanding the distinction between mastery measurement and general outcome measurement. Mastery measurement evaluates student performance on a specific skill within a narrow instructional sequence, whereas general outcome measurement, often using multiskill curriculum-based measures, tracks performance on a broader sample of grade-level content over time (Rojo et al., 2021;

VanDerHeyden et al., 2017). Each approach serves a distinct purpose. Mastery measurement is most useful for tracking the acquisition of a targeted skill during intervention, whereas multiskill CBM provides an indicator of broader mathematics growth and is better suited for evaluating responsiveness to intervention over a longer instructional period (Rojo et al., 2021; VanDerHeyden et al., 2017). Once data are collected, educators must be able to interpret time-series graphs accurately by evaluating the level, trend, and variability of student performance relative to an established goal line to determine whether the current instructional plan is producing adequate progress or whether intensification is warranted (Peltier et al., 2021).

Collaborative teams conduct systematic data reviews to inform instructional decisions, ensuring that student placement within tiers reflects current performance rather than static labels (Center on Multi-Tiered Systems of Support, 2022; Eagle et al., 2014). Implementation fidelity is critical to MTSS effectiveness at every level, including fidelity of intervention delivery, progress monitoring administration, and the data-based decision-making process itself. Schools demonstrating greater adherence to MTSS structures and practices exhibit stronger academic and behavioral outcomes (Scott et al., 2019). By integrating high-quality instruction, early identification, systematic data use, and consistent fidelity evaluation, MTSS creates a coherent framework for identifying mathematics difficulties early and responding before they become more persistent and intensive.

6.2. Core Components of Internally Consistent Mathematics IEPs

Individualized Education Programs (IEPs) serve as the mechanism through which empirically validated mathematics practices are translated into legally binding, student-specific instructional plans. An internally consistent mathematics IEP is the cornerstone of a student's right to a free appropriate public education; when present levels, goals, services, and progress monitoring are misaligned, the IEP ceases to function as a coherent instructional blueprint and becomes both educationally ineffective and legally vulnerable (Yell et al., 2020; Zirkel & Hetrick, 2016). The Individuals with Disabilities Education Act (IDEA) requires that IEPs include a clear statement of present levels of academic achievement and functional performance (PLAAFP), measurable annual goals, a description of special education and related services, and a plan for monitoring progress, with these components operating as an integrated system rather than as isolated compliance elements (Yell et al., 2020).

Internally consistent mathematics IEPs begin with data-based PLAAFP statements that precisely describe the student's current mathematics performance and identify specific strengths and instructional needs within mathematical domains (Ruble et al., 2010). Each identified need is addressed through measurable annual goals written using the SMART framework—goals that are **S**pecific to the mathematical skill or concept, **M**easurable with clear criteria and assessment methods, **A**mbitious rather than merely achievable (as mandated by the Supreme Court in *Endrew F. v. Douglas County School District,* 2017.), **R**elevant to grade-level content standards, and **T**ime-bound within the annual IEP cycle (*Endrew F. v. Douglas County School District,* 2017; Yell et al., 2020). The emphasis on ambitious goals reflects the legal requirement that IEPs be reasonably calculated to enable students to make progress appropriate to their circumstances, challenging students to reach beyond minimal competency toward meaningful advancement in mathematics (*Endrew F. v. Douglas County School District,* 2017).

Specially designed instruction, the instructional adaptations and strategies tailored to address a student's disability-related needs, must be clearly specified in the IEP and aligned with the ambitious annual goals (IDEA, 2004; Rodgers et al., 2021; Yell et al., 2020). In content areas such as mathematics, instruction should include empirically supported practices, such as systematic, explicit instruction, along with appropriate accommodations and related services (Witzel et al., 2026). To ensure that specially designed instruction produces the intended effects, IEPs must also specify how student progress toward annual goals will be measured and reported (IDEA, 2004). These progress-monitoring procedures specify the assessment tools, measurement frequency, and criteria used to determine whether the student is achieving the ambitious progress envisioned by the IEP team through the delivery of specially designed instruction (Fuchs et al., 2021; Yell et al., 2020). Research and legal analyses indicate that IEPs are most vulnerable when goals are vague, services are disconnected from identified needs, or progress monitoring is absent or inadequately specified (Ruble et al., 2010; Zirkel & Hetrick, 2016). In mathematics interventions, internal consistency ensures that instructional decisions are data-driven, targeted to specific mathematical constructs, and systematically evaluated, thereby supporting both meaningful educational progress and legal defensibility (Yell et al., 2020).

6.3. Core Components of Data-Based Individualization

While data-based decision making is an integral part of any MTSS framework and helps make decisions about student movement across tiers, data-based individualization (DBI) is used within Tier 2 or Tier 3 intervention to make timely decisions about whether the intervention efforts are on track to meet the individual needs of students (Powell et al., 2024). DBI has shown promise in improving students' academic outcomes (Jung et al., 2018). DBI starts with an instructional platform constructed of an empirically validated intervention or set of strategies. Alongside instruction, teachers or tutors collect frequent progress-monitoring data. After several data points, the data is analyzed to determine whether students are on track to meet their mathematics goals. If students are making adequate progress, then the intervention efforts continue. If students are not making adequate progress, diagnostic data are collected and analyzed to inform adaptations to the instructional platform (Powell et al., 2024; Weingarten et al., 2023). With the adaptations in place, teachers or tutors continue to collect progress-monitoring data and examine it to determine whether students are on track or need further adaptations.

Conclusion

This IC has synthesized the research and policy context, foundational learning frameworks, content knowledge requirements, and empirically validated instructional practices that collectively define high-quality mathematics instruction for students with disabilities and mathematics difficulties in grades K-5. The narrative descriptions accompanying the IC matrix are intended to provide SEAs, IHEs, and other stakeholders with the substantive grounding necessary to evaluate and strengthen current practices across three domains: state licensure requirements, professional development activities at the state, district, and school levels, and teacher preparation programs serving students with and at risk for mathematics difficulties.

The evidence base described in this document is not uniform in depth or breadth. Some practices, such as systematic explicit instruction, representations, and number line instruction, are supported by extensive empirically based literature spanning multiple decades and diverse student populations. Others, such as mathematical language instruction and word-problem solving, reflect growing but still developing bodies of evidence. The IC matrix in the Appendix indicates the relative strength of evidence for each component, and stakeholders are encouraged to consult that documentation alongside this narrative.

Ultimately, the purpose of this IC is practical: to translate a complex and multidisciplinary research base into actionable guidance that teacher preparation programs, professional developers, and policymakers can use to set goals, design improvement plans, and establish benchmarks for evaluating progress. This IC is intended to be used flexibly and in conjunction with other resources to maximize its benefit for programs. Teacher preparation programs that do not offer a dedicated mathematics course may distribute the essential components across multiple courses or field-based experiences, while still prioritizing the empirically validated practices in Section 5.0 as the core instructional content. Federally supported resources, such as the National Center on Intensive Intervention, provide complementary tools, including reviews of intervention and progress-monitoring measures and professional development materials, that can extend and support the implementation of the practices described in this IC. Students with mathematics difficulties deserve instruction that is grounded in evidence, delivered with fidelity, and responsive to their individual learning needs. Such instruction is essential not only for addressing persistent skill gaps but also for ensuring that students can access grade-level and increasingly advanced mathematics content. This IC is intended to help systems move that commitment from principle to practice.

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Appendix A: Essential Components for Empirically Validated Mathematics Instruction for Grades K-5

Essential Components	Implementation Levels				
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria level from 0 to 3. Score and rate each item separately.	**Level 0**	**Level 1**	**Level 2**	**Level 3**	**Rating**
	There is no evidence that the component is included in the syllabus, or the syllabus only mentions the component.	Must contain at least one of the following: reading, test, lecture/ presentation, discussion, modeling/ demonstration, or quiz.	Must contain at least one item from Level 1, plus at least one of the following: observation, project/activity, case study, or lesson plan study.	Must contain at least one item from Level 1 as well as at least one item from Level 2, plus at least one of the following: tutoring, small group student teaching, or whole group internship.	Rate each item as the number of the highest variation receiving an X under it.
1.0 Influences on Mathematics Policy and Practice in the United States					
1.1 The concept of empirically validated practices to teach mathematics (and specially designed instruction [SDI] in mathematics) and the fields with significant contributions toward the science (i.e., psychology, mathematics, neuroscience, and education) 1.2 History of mathematics instruction in the United States, including the "math wars" and the major empirical and theoretical influences on practice (e.g., constructivism, explicit instruction) 1.3 Recommendations contained in important syntheses of evidence on mathematics instruction					

(e.g., *Adding It Up* (Kilpatrick et al., 2001); National Mathematics Advisory Panel, 2008; Fuchs et al., 2021). 1.4 Recommendations contained in relevant practice guides from the U.S. Department of Education's Institute of Education Sciences (e.g., Frye et al., 2013; Fuchs et al., 2021; Siegler et al., 2010; Star et al., 2019; Woodward et al., 2018) 1.5 Federal policies that affect mathematics instruction and intervention (e.g., No Child Left Behind [NCLB]; Every Student Succeeds Act [ESSA]; Individuals with Disabilities Education Improvement Act [IDEA]) 1.6 Nationwide initiatives that affect mathematics instruction and intervention (e.g., Common Core State Standards [CCSS]) 1.7 Standards related to mathematics instruction and intervention that have been put forth by professional organizations (e.g., Council for Exceptional Children [CEC], National Council for Teachers of Mathematics [NCTM]) 1.8 Influence of public media in mathematics policy and practice					

Essential Components	Implementation Levels				
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria level from 0 to 3. Score and rate each item separately.	**Level 0**	**Level 1**	**Level 2**	**Level 3**	**Rating**
	There is no evidence that the component is included in the syllabus, or the syllabus only mentions the component.	Must contain at least one of the following: reading, test, lecture/ presentation, discussion, modeling/ demonstration, or quiz.	Must contain at least one item from Level 1, plus at least one of the following: observation, project/activity, case study, or lesson plan study.	Must contain at least one item from Level 1 as well as at least one item from Level 2, plus at least one of the following: tutoring, small group student teaching, or whole group internship.	Rate each item as the number of the highest variation receiving an X under it.
2.0 Framework for the Teaching and Learning of Mathematics					
2.1 Emphasis on the cumulative nature of mathematics and common pathways for learning mathematics, with particular focus on the arithmetic to algebra gap 2.2 Characteristics of learners with mathematics difficulty and/or disability 2.3 Five essential components of mathematics based on *Adding it Up* (including the flaws of this conceptualization of mathematics) 2.4 Theoretical and conceptual frameworks for understanding how students learn mathematics					

2.5 Instructional hierarchy for teaching mathematics (i.e., acquisition, proficiency, generalization, adaptation)					
Essential Components	**Implementation Levels**				
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria level from 0 to 3. Score and rate each item separately.	**Level 0**	**Level 1**	**Level 2**	**Level 3**	**Rating**
	There is no evidence that the component is included in the syllabus, or the syllabus only mentions the component.	Must contain at least one of the following: reading, test, lecture/ presentation, discussion, modeling/ demonstration, or quiz.	Must contain at least one item from Level 1, plus at least one of the following: observation, project/activity, case study, or lesson plan study.	Must contain at least one item from Level 1 as well as at least one item from Level 2, plus at least one of the following: tutoring, small group student teaching, or whole group internship.	Rate each item as the number of the highest variation receiving an X under it.
3.0 Fundamentals of Mathematics (K-2)					
3.1 Counting and cardinality - Stable order (i.e., knowing number names in order) - One-to-one correspondence (i.e., counting objects) - Cardinality (i.e., the last number count shows the total in a set) - Compare numbers 3.2 Operations and algebraic thinking - Addition as putting together and adding on - Subtraction as taking away and comparing - Relationship between addition and subtraction - Add and subtract within 20 - Word problems with addition and subtraction - Understanding of the symbols of mathematics					

3.3 Number and operations in base 10 - Understanding numbers 11 to 19 - Understanding tens and ones - Understanding hundreds, tens, and ones - Using place value to add and subtract 3.4 Measurement and data - Describe measurable attributes - Classify and count within categories - Measuring and iterating lengths - Telling and writing time - Working with money - Interpreting data 3.5 Geometry - Two-dimensional shapes - Three-dimensional shapes					

Essential Components	Implementation Levels				
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria level from 0 to 3. Score and rate each item separately.	**Level 0**	**Level 1**	**Level 2**	**Level 3**	**Rating**
	There is no evidence that the component is included in the syllabus, or the syllabus only mentions the component.	Must contain at least one of the following: reading, test, lecture/ presentation, discussion, modeling/ demonstration, or quiz.	Must contain at least one item from Level 1, plus at least one of the following: observation, project/activity, case study, or lesson plan study.	Must contain at least one item from Level 1 as well as at least one item from Level 2, plus at least one of the following: tutoring, small group student teaching, or whole group internship.	Rate each item as the number of the highest variation receiving an X under it.
4.0 Fundamentals of Mathematics (3-5)					
4.1 Operations and algebraic thinking - Multiplication as equal groups, arrays, area, and multiplicative comparison - Division as equal groups, arrays, area, and multiplicative comparison - Relationship between multiplication and division - Multiply and divide within 100 - Factors and multiples 4.2 Number and operations in base 10 - Multi-digit whole-number computation with all four operations - Understanding thousands, hundreds, tens, and ones - Multi-digit decimal computation with all four operations 4.3 Number and operations with fractions - Understand fractions as numbers					

- Fraction equivalence - Fraction comparing and ordering - Computation with fractions - Decimal notation for fractions 4.4 Measurement and data - Solve problems about time, liquid volume, and masses of objects - Interpret data - Area and perimeter - Conversions of units - Measurement of angles - Volume 4.5 Geometry - Reason with shapes and their attributes - Lines - Angles - Coordinate planes					

Essential Components	Implementation Levels				
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria level from 0 to 3. Score and rate each item separately.	**Level 0**	**Level 1**	**Level 2**	**Level 3**	**Rating**
	There is no evidence that the component is included in the syllabus, or the syllabus only mentions the component.	Must contain at least one of the following: reading, test, lecture/ presentation, discussion, modeling/ demonstration, or quiz.	Must contain at least one item from Level 1, plus at least one of the following: observation, project/activity, case study, or lesson plan study.	Must contain at least one item from Level 1 as well as at least one item from Level 2, plus at least one of the following: tutoring, small group student teaching, or whole group internship.	Rate each item as the number of the highest variation receiving an X under it.
5.0 Empirically-Validated Practices for Teaching and Learning the Fundamentals of Mathematics					
5.1 *Systematic Instruction-Provide systematic instruction during intervention to develop student understanding of mathematical ideas.* **-Sequential Content Design:** Understanding how to build mathematics concepts incrementally, ensuring each lesson connects to prior learning and builds toward clear learning outcomes **-Review and Integration:** Strategies for systematically reviewing previously learned content while introducing new material, including mixing problem types to prevent overgeneralization **-Accessible Number Selection:** Using simple, manageable numbers when introducing new concepts so students can focus on understanding the concept rather than complex calculations **-Visual and Verbal Supports:** Implementing prompts, questions, gestures, and visual aids to help students attend to and remember connections between concepts **-Immediate Feedback Techniques:** Identifying					

student misunderstandings through questioning and providing corrective feedback that helps students self-identify where their thinking went wrong **-Pacing for Mastery**: Understanding that students needing intervention require more time and repeated exposure to develop a deep understanding of critical grade-level topics 5.2 *Mathematical Language-Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.* **-Precise Vocabulary Instruction:** Teaching student-friendly definitions of mathematical terms (e.g., "equal," "greater than," "addend") and avoiding informal language like "flip-flop property" **-Contextual Word Introduction:** Introducing new vocabulary during instruction with concrete examples, representations, and gestures to provide meaningful context **-Consistent Mathematical Language Use:** Modeling and consistently using correct mathematical terminology throughout all lessons, discussions, and feedback **-School-Wide Vocabulary Alignment:** Developing shared lists of mathematical terminology used consistently across grade levels and settings to support student learning **-Student Explanation Support:** Providing sentence starters, guiding questions, and visual supports (like word walls) to help students use mathematical language in verbal and written explanations **-Multiple Exposure Strategy:** Understanding that students in intervention need multiple encounters with mathematical vocabulary across various contexts to develop a deep understanding					

5.3 *Representations-Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.* **-Intentional Representation Selection:** Choosing appropriate concrete (3D manipulatives like base 10 blocks, counters) and semi-concrete (2D drawings, diagrams) representations that accurately model specific concepts **-Proportional Representations:** Using manipulatives where relationships are proportional (e.g., base 10 blocks where 10s are actually 10 times the size of ones) for place value and fractions **-Connecting Multiple Representations:** Explicitly linking concrete representations to semi-concrete representations and to abstract mathematical notation simultaneously during instruction **-Representations as Thinking Tools:** Providing extensive practice opportunities for students to use representations independently to model and solve problems, not just observe teacher demonstrations **-Strategic Fading:** Understanding when and how to gradually reduce emphasis on concrete/semi-concrete representations as students demonstrate understanding with abstract notation **-Consistent Representation Use**: Maintaining the same core set of representations across lessons, units, and grade levels to reinforce learning and avoid confusion 5.4 *Number Lines-Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics*. **-Concrete to Abstract Progression:** Starting with physical number paths students can walk on, then connecting to number lines on paper to build					

understanding of unit length and distance **-Number Line Fundamentals:** Teaching basic characteristics, including equidistant intervals, predictable sequences, arrows showing infinity, and how numbers increase/decrease **-Magnitude Understanding:** Using number lines to help students compare whole numbers (0-10, 0-100) by understanding that greater numbers are further to the right from zero **-Operations on Number Lines:** Demonstrating addition as moving right (combining) and subtraction as moving left (taking away) using the distance between numbers **-Benchmark Number Strategies:** Teaching students to use 0, 5, and 10 as reference points for estimating and comparing the magnitude of other numbers **-Multiple Representations Connection:** Linking physical manipulatives (like connecting cubes) to number line representations to reinforce one-to-one correspondence and counting concepts 5.5 *Fluency - Build students' fluency, meaning both accuracy and efficiency, with foundational mathematics skills and concepts through intentionally designed, daily practice with feedback.* **-Accuracy Before Efficiency:** Grounding fluency in Haring and Eaton's (1978) instructional hierarchy, establishing accuracy in a skill before building efficiency and ease of execution **-Fluency Across Content:** Recognizing that fluency applies across content and grade levels, from counting and the four operations to fractions, decimals, measurement, and algebra **-Intentionally Designed Practice:** Planning deliberate, targeted practice opportunities rather than					

relying on incidental exposure, so students rehearse the specific skills being developed -**Practice With Feedback:** Pairing practice with timely feedback so students confirm correct responses and correct errors before they become habituated **-Daily Fluency Building:** Incorporating brief fluency-building activities into mathematics instruction or intervention as a routine daily component of the lesson **-Increased Practice for Students with Difficulty:** Understanding that students with mathematics difficulty require many more practice opportunities to reach fluency, and planning for that larger dosage **-Timed Activities:** Regularly include timed activities as one way to build students' fluency in mathematics. ○ **Strategic Fluency Focus:** Selecting specific subtasks for fluency development (e.g., addition facts to 10, counting by 5s) that support broader mathematical learning goals ○ **Strategy-Based Practice:** Teaching efficient strategies (e.g., counting on from the larger number, using doubles) before engaging in timed activities, never emphasizing speed over accuracy ○ **Brief, Focused Activities:** Implementing 1-5 minute fluency activities as one component within broader lessons, using flashcards, worksheets, or digital tools ○ **Progress Monitoring:** Having students track and graph their own scores over time, setting goals to "meet or beat" previous scores to maintain motivation ○ **Immediate Corrective Feedback:** Providing instant feedback on errors and prompting students to correct mistakes using taught strategies before moving to the next problem					

○ **Reducing Math Anxiety:** Understanding that accuracy is more important than speed, minimizing competition, allowing students to work as groups, and ensuring students know worksheets contain more problems than expected to be completed 5.6 *Word Problem Solving-Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.* **-Problem Type Identification:** Teaching students to recognize word problem types based on underlying mathematical structure (Change, Combine, Compare, Equal Groups) rather than key words **-Solution Method Instruction:** Providing explicit instruction in solution strategies specific to each problem type, using worked examples and think-alouds to model the problem-solving process **-Concrete Problem Visualization:** Using counters, role-playing, and simple drawings to help students visualize problem situations and identify relevant information **-Mathematical Vocabulary in Context:** Pre-teaching words that may be difficult (comparison words like "more/fewer," category words like "pets") before students encounter them in problems **-Graduated Problem Complexity:** Starting with problems containing all quantities, then introducing unknown quantities, then varying which quantity is unknown, and finally adding irrelevant information **-Problem Type Discrimination:** Mixing previously learned and new problem types throughout instruction to help students understand when to apply different strategies and avoid pattern-based solving					

5.7 *Needs of Dual Language Learners — Support English learners with mathematics difficulties through linguistically responsive instructional practices that reduce unnecessary linguistic load while preserving mathematical demand.* **-Linguistic Demand Analysis:** Identifying vocabulary and relational language embedded in word problems and mathematical tasks that may create barriers for English learners **-Explicit Vocabulary and Language Instruction:** Pre-teaching mathematical vocabulary and academic language before introducing problem-solving tasks **-Visual and Representational Scaffolding:** Selecting and using concrete manipulatives, diagrams, and visual models to provide access to mathematical content independent of language proficiency **-Assessment in Home Language and English:** Understanding the value of collecting progress monitoring data in both the home language and English **-Intervention Design for English Learners:** Understanding that mathematics intervention for English learners should prioritize focused mathematics instruction with linguistic scaffolds as supports 5.8 *Student expression in mathematics: Verbalizations and writing-Support students in developing mathematical understanding through expressive practices that require them to articulate, explain, and justify their reasoning. Student verbalizations, including self-explanation and peer-to-peer mathematical discourse, and writing within mathematics are empirically supported practices that deepen conceptual understanding and strengthen problem-solving performance across student*					

populations, including students with mathematics difficulties. - **Self-explanation and prompted verbalization**: Encourage students to articulate their mathematical reasoning aloud during problem solving, including explaining solution steps, justifying decisions, and connecting procedures to underlying concepts. Prompting students to self-explain, rather than simply practice, produces stronger and more durable learning outcomes. - **Peer-to-peer mathematical discourse:** Structure opportunities for students to explain their mathematical thinking to peers and to evaluate the reasoning of others. Peer explanation supports both the explainer and the listener by requiring students to organize and communicate mathematical ideas in their own words. **-Writing to explain mathematical reasoning**: Incorporate writing as a regular instructional practice within mathematics, including written explanations of problem-solving processes, justification of solutions, and reflection on mathematical concepts. Writing about mathematics reliably enhances learning across grade levels and content areas. -**Mathematical journals and written expression:** Use mathematics journals and structured writing tasks as tools for consolidating conceptual understanding and monitoring student thinking over time. These practices support metacognitive development alongside content learning. -**Embedded writing instruction for students with disabilities:** For students with mathematics difficulties, provide explicit instruction in how to write mathematical explanations rather than assuming this skill transfers automatically. Embedding structured writing supports within mathematics					

intervention strengthens both mathematical reasoning and written expression simultaneously.					
Essential Components	**Implementation Levels**				
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria level from 0 to 3. Score and rate each item separately.	**Level 0**	**Level 1**	**Level 2**	**Level 3**	**Rating**
	There is no evidence that the component is included in the syllabus, or the syllabus only mentions the component.	Must contain at least one of the following: reading, test, lecture/ presentation, discussion, modeling/ demonstration, or quiz.	Must contain at least one item from Level 1, plus at least one of the following: observation, project/activity, case study, or lesson plan study.	Must contain at least one item from Level 1 as well as at least one item from Level 2, plus at least one of the following: tutoring, small group student teaching, or whole group internship.	Rate each item as the number of the highest variation receiving an X under it.
6.0 Characteristics of Effective Assessment, Instruction, and Intervention in Mathematics					
6.1 Core components of an effective multi-tiered system of supports (MTSS) 6.2 Core components of Internally consistent Math IEPs 6.3 Core components of Data-Based Individualization (DBI)					

Appendix B: Levels of Evidence for Empirically Validated Mathematics Instruction for Grades K-5.

Note: Evidence level designations reflect the strength of research supporting each instructional practice or knowledge domain for improving student outcomes in mathematics. Meta-analytic evidence was applied toward group experimental and quasi-experimental design criteria per CEEDAR Center guidance. Evidence designations for Sections 1.0 through 4.0 reflect the extent to which these knowledge components are grounded in peer-reviewed literature, professional standards, and policy documents rather than experimental intervention research.

Essential Components	Level of Evidence	Supporting References
1.0 Influences on Mathematics Policy and Practice in the United States		
1.1 The concept of empirically validated mathematics instruction, including specially designed instruction (SDI), and the interdisciplinary contributions of psychology, mathematics, neuroscience, and education	Emerging	Cook et al. (2015); Sayeski et al. (2022); Witzel et al. (2024); Witzel et al. (2026)
1.2 History of mathematics instruction in the United States, including the "math wars" and the major empirical and theoretical influences on practice	Emerging	Gersten (2025); Kilpatrick et al. (2001); Morgan et al. (2015); NCTM & CEC (2024); Powell et al. (2025); Witzel et al. (2024)
1.3 Recommendations contained in important syntheses of evidence on mathematics instruction	Emerging	National Mathematics Advisory Panel (2008); National Research Council (2001); Fuchs et al. (2021); Powell et al. (2025)
1.4 Recommendations contained in relevant practice guides from IES	Emerging	Fuchs et al. (2021); Frye et al. (2013); Gersten et al. (2009a); Siegler et al. (2010); Woodward et al. (2012)
1.5 Federal policies that affect mathematics instruction and intervention	Emerging	IDEA (2004); ESSA (2015); No Child Left Behind Act (2002)

Essential Components	Level of Evidence	Supporting References
1.6 Nationwide initiatives that affect mathematics instruction and intervention	Emerging	NGA & CCSSO (2010); NCTM (1989); NCTM (2000)
1.7 Standards related to mathematics instruction and intervention put forth by professional organizations	Emerging	Aceves & Kennedy (2024); McLeskey et al. (2017); Powell et al. (2025); NCTM & CEC (2024)
1.8 Influence of public media in mathematics policy and practice	Emerging	NCES (2026); OECD (2023); von Davier et al. (2024)
2.0 Foundations for Mathematics Teaching and Learning		
2.1 Emphasis on the cumulative nature of mathematics and common pathways for learning mathematics, with particular focus on the arithmetic to algebra gap	Moderate	Duncan et al. (2007); Hughes et al. (2014); Ketterlin-Geller et al. (2007); Ketterlin-Geller & Chard (2011); Pillay et al. (1998); Siegler et al. (2012); Witzel (2016)
2.2 Characteristics of learners with mathematics difficulty and/or disability	Moderate	Cirino et al. (2015); Geary (2013); Peng et al. (2018); Peng et al. (2020)
2.3 Five essential components of mathematics proficiency based on Adding It Up, including a critical examination of the Math Rope conceptualization	Moderate	National Research Council (2001); Fuchs et al. (2021); Rittle-Johnson et al. (2015); Powell et al. (2025)
2.4 Theoretical and conceptual frameworks for understanding how students learn mathematics	Emerging	Kirschner et al. (2006); Sweller et al. (2011); Vygotsky (1978); Piaget (1952)
2.5 Instructional hierarchy for teaching mathematics (i.e., acquisition, proficiency, generalization, adaptation)	Emerging	Haring & Eaton (1978); Ardoin & Daly (2007); Burns et al. (2015); Rohrer et al. (2020)

Essential Components	Level of Evidence	Supporting References
3.0 Fundamentals of Mathematics (Grades K-2)		
3.1 Counting and cardinality	Moderate	Aunio et al. (2015); Geary et al. (2018); Jordan et al. (2010); NGA & CCSSO (2010)
3.2 Operations and algebraic thinking	Moderate	Bryant et al. (2008); Fuchs et al. (2021); Powell et al. (2016); NGA & CCSSO (2010)
3.3 Number and operations in base 10	Moderate	Fuchs et al. (2021); Geary (2011); NGA & CCSSO (2010)
3.4 Measurement and data	Emerging	Doabler et al. (2022); Frye et al. (2013); NGA & CCSSO (2010)
3.5 Geometry	Emerging	Gavin et al. (2013); NGA & CCSSO (2010)
4.0 Fundamentals of Mathematics (Grades 3-5)		
4.1 Operations and algebraic thinking	Moderate	Fuchs et al. (2021); NGA & CCSSO (2010); Siegler et al. (2010)
4.2 Number and operations in base 10	Moderate	Fuchs et al. (2021); NGA & CCSSO (2010)
4.3 Number and operations with fractions	Moderate	Siegler et al. (2010); Fuchs et al. (2016); Shin & Bryant (2015); Jayanthi et al. (2021)
4.4 Measurement and data	Emerging	NGA & CCSSO (2010); Woodward et al. (2012)
4.5 Geometry	Emerging	NGA & CCSSO (2010)
5.0 Empirically Validated Practices for Teaching the Fundamentals of Mathematics		
5.1 Systematic, explicit instruction	Strong	Dennis et al. (2016); Fuchs et al. (2021); Gersten et al. (2009b); Myers et al. (2021); Stockard et al. (2018)

Essential Components	Level of Evidence	Supporting References
5.2 Mathematical language	Strong	Stevens et al. (2023); Lariviere et al. (2024); Turan & De Smedt (2022); Nelson & Kiss (2021)
5.3 Representations (concrete-representational-abstract framework)	Strong	Ebner et al. (2025); Peltier et al. (2019); Witzel et al. (2003)
5.4 Number lines	Strong	Barbieri et al. (2020); Fuchs et al. (2021); Rojo et al. (2023)
5.5 Fluency	Strong	Burns et al. (2010); Codding et al. (2011); Douglas et al. (2026)
5.6 Word Problem Solving	Strong	Myers et al. (2022b); Myers et al. (2023); Lein et al. (2020); Shin et al. (2021)
5.7 Needs of dual language learners	Moderate	Lei et al. (2020); Driver & Powell (2017); Orosco et al. (2013); Kong & Swanson (2019)
5.8 Student expression in mathematics: Verbalizations and writing	Moderate	Graham et al. (2020); Rittle-Johnson et al. (2017); Riccomini et al. (2024)
6.0 Characteristics of Effective Assessment, Instruction, and Intervention in Mathematics		
6.1 Core components of an effective multi-tiered system of supports (MTSS)	Moderate	Fuchs & Fuchs (2017); Scott et al. (2019); Choi et al. (2020)
6.2 Core components of internally consistent mathematics IEPs	Emerging	Endrew F. v. Douglas County School District (2017); Yell & Stecker (2003); Yell et al. (2020)
6.3 Core components of data-based individualization (DBI)	Emerging	Fuchs et al. (2016); Powell et al. (2024); Weingarten et al. (2023)