Handout #3

Concrete-to-Representational-to-Abstract (C-R-A) Instruction

What is the purpose of C-R-A instruction?

The purpose of teaching through a C-R-A sequence of instruction is to ensure students develop a concrete understanding of the math concepts/skills they learn. Having students represent their concrete (representational) understandings by drawing simple pictures that replicate or mimic their use of concrete materials provides students a supported process for transferring their concrete understanding to the abstract level. Moreover, teaching students how to draw solutions in problem-solving situations provides an excellent strategy for problem solving in the future.

What is CRA instruction?

Concrete – Each math concept/skill is first modeled with concrete materials (e.g., chips, unifix cubes, base-ten blocks, beans and bean sticks, pattern blocks). Students are provided many opportunities to practice and demonstrate mastery using concrete materials.

Representational – The math concept/skill is next modeled at the representational (semi-concrete) level, which involves drawing pictures that represent the concrete objects previously used (e.g., tallies, dots, circles, stamps that imprint pictures for counting). Students are provided many opportunities to practice and demonstrate mastery by drawing solutions.

Abstract – The math concept/skill is finally modeled at the abstract level (using only numbers and mathematical symbols). Students are provided many opportunities to practice and demonstrate mastery at the abstract level before moving to a new math concept/skill.

As a teacher moves through a concrete-to-representational-to-abstract sequence of instruction, the abstract numbers and/or symbols should be used in conjunction with the concrete materials and representational drawings. *This is especially important for students with special needs, since it promotes association of abstract symbols with their concrete and representational understandings.*

Important considerations when implementing C-R-A instruction

1. First use appropriate concrete objects (e.g., counting objects such as beans, chips, unifix cubes, and popsicle sticks) to teach particular math concepts/skills. Base-ten materials are excellent for building understanding of place value and other number and number sense relationships.

2. After students demonstrate mastery at the concrete level (e.g., 5 out of 5 correct for three consecutive days), teach appropriate drawing techniques where students problem-solve by

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drawing simple representations of the concrete objects they previously used (e.g., tallies, dots, and circles). By replicating the movements students previously used with concrete materials, drawing simple representations of those objects supports students' evolving abstract understanding of the concept/skill.

3. After students demonstrate mastery at the representational level (e.g., 10 out of 10 correct for three consecutive days) use appropriate strategies for assisting students to move to the abstract level of understanding for a particular math concept/skill. For students who have a concrete level of understanding, provide opportunities to use their language to describe their solutions and their understanding of the mathematics concept/skill they are learning.

Students with special needs often have difficulty developing abstract-level understandings. Several barriers can make this situation occur. Sometimes students have never developed conceptual understanding of the target mathematics concept/skill. Typically this occurs when students have not been allowed to develop that understanding at the concrete & representational levels of understanding. Two ways to manage this situation are to reteach the mathematics concept/skill using appropriate concrete materials and then explicitly show the relationship between the concrete materials and the abstract representation of the materials. Other possible reasons students may have difficulty developing abstract understandings of a particular mathematics concept/skill: they have difficulty with basic facts because of memory problems; they repeat procedural mistakes that can result from perceptual processing deficits, attention difficulties, or memory problems; or they use faulty algorithms that result from not understanding prerequisite concepts/skills (e.g., place value).

How do I implement C-R-A instruction?

- When initially teaching a math concept/skill, describe and model it using concrete objects (concrete level of understanding) and provide students multiple practice opportunities using concrete objects.
- When students demonstrate mastery by using concrete objects, describe and model how to perform the skill by drawing or using pictures that represent concrete objects (representational level of understanding). One again, provide multiple practice opportunities where students draw their solutions or use pictures to problem-solve.
- When students demonstrate mastery by drawing solutions, describe and model how to perform the skill using only numbers and math symbols (abstract level of understanding). Provide multiple opportunities for students to practice performing the skill using only numbers and symbols.
- After students master performing the skill at the abstract level of understanding, ensure students maintain their skill level by providing periodic practice opportunities for the math skills.

How does C-R-A help students who have learning problems?

- By linking learning experiences from concrete, to representational, to abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections.
- CRA blends conceptual and procedural understanding in a structured way so that students learn both the "How" and the "Why" of the problem-solving procedures they learn, and they learn the

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"What"— they develop conceptual understanding of the mathematics concept that underlies the problem-solving process.

Additional Resources

http://www.coedu.usf.edu/main/departments/sped/mathvids/index.html.

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